Inspecting identities

Problem

Take a look at these identities.

✱

$$\cos^2\frac{\theta}{2} \equiv \frac{1}{2}(1+\cos\theta) \qquad \sin^2\frac{\theta}{2} \equiv \frac{1}{2}(1-\cos\theta)$$

How could you use these identities to help you sketch graphs of $y = \cos^2\frac{\theta}{2}$ and $y = \sin^2\frac{\theta}{2}$?

Inspecting identities

Things you might have noticed

Take a look at these identities.

$$\cos^2\frac{\theta}{2} \equiv \frac{1}{2}(1+\cos\theta) \qquad \sin^2\frac{\theta}{2} \equiv \frac{1}{2}(1-\cos\theta)$$

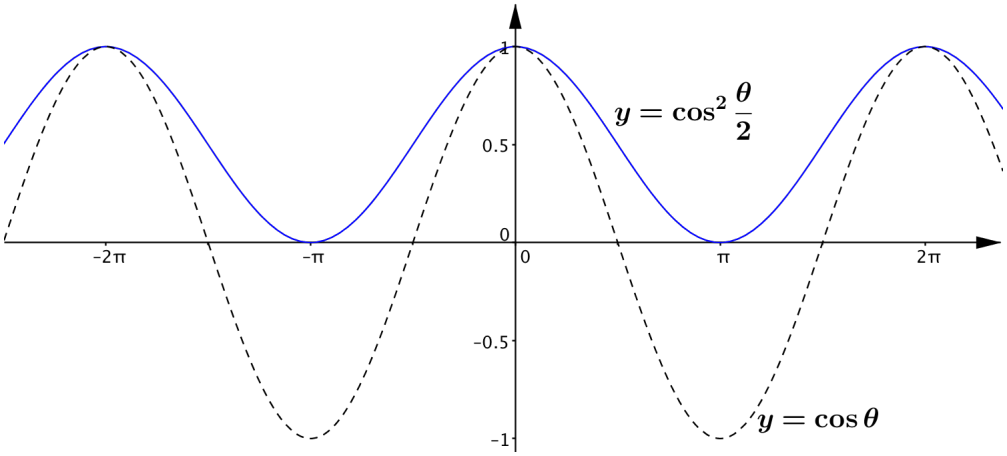
How could you use these identities to help you sketch graphs of $y = \cos^2\frac{\theta}{2}$ and $y = \sin^2\frac{\theta}{2}$?

These ideas may help you to sketch the graphs of $y = \cos^2\frac{\theta}{2}$ and $y = \sin^2\frac{\theta}{2}$

- As the left-hand side is a square in each case, we know the right hand sides must be zero or positive. This is fine, because $-1 \leq \cos\theta \leq 1$, so $1+\cos\theta$ will be at least 0 and at most 2. Similarly, $1-\cos\theta$ must be between 0 and 2.
- The right-hand sides of the identities are the same except for a minus sign, so we'd expect the behaviour of the two functions to be very similar, but the maximum and minimum values will occur at different values of θ. For example, $\frac{1}{2}(1+\cos\theta)$ will be 1 when $\theta = \ldots, -4\pi, -2\pi, 0, 2\pi, 4\pi, \ldots$ (i.e. at even multiples of π) whereas $\frac{1}{2}(1-\cos\theta)$ will be 1 when $\theta = \ldots, -5\pi, -3\pi, -\pi, \pi, 3\pi, 5\pi, \ldots$ (i.e. at odd multiples of π).
- The right-hand side of each identity only involves $\cos\theta$ and constants, so the graphs of $y = \cos^2\frac{\theta}{2}$ and $y = \sin^2\frac{\theta}{2}$ can be thought of as transformations of a $y = \cos\theta$ graph. Which transformations are involved in each case? What is the frequency of $y = \cos^2\frac{\theta}{2}$ and $y = \sin^2\frac{\theta}{2}$?
- Again, from the right-hand side, the identities tell us that $\cos^2\frac{\theta}{2}$ is the mean of 1 and $\cos\theta$ and $\sin^2\frac{\theta}{2}$ is the mean of 1 and $-\cos\theta$. What does this tell you about the graph of $y = \cos^2\frac{\theta}{2}$ in relation to the graphs of $y = 1$ and $y = \cos\theta$?

Graphs

Here are the graphs of $y = \cos^2\frac{\theta}{2}$ and $y = \cos\theta$.



Here are the graphs of $y = \sin^2 \frac{\theta}{2}$ and $y = \cos\theta$.