Two-way calculus

Problem

Each column and row heading in the following table is a property that a function may or may not have. A function can appear in a cell if it has the properties in the corresponding row and column.

We have omitted some headings, and some entries in cells. Can you complete the table?

You might find it helpful to draw some sketches. You could use graph-sketching software such as Desmos to help you, but try to do the sketching by hand before reaching for a computer or calculator!

Make sure that you can explain why each function has the desired properties.

	The curve is …creasing for $x > 1$	**Has a local …imum with** y**-coordinate** 1	
Has a stationary point at $(1, 1)$	$y = x^3 + 3x^2 - 9x + 6$		$y = 3x^4 - 4x^3 + 2$
		$y = 2x^3 + 9x^2 + 1$	$y = 1 - \frac{1}{4}x^4 - x^3$
Has an … number of stationary points			$y = x^5 - x^3 + 5$

- Can you complete the table using a different function in every cell?
- On the other hand, can you complete the table with a smaller number of functions?
- Did you have any choice about the column and row headings?

Have you tried sketching the functions we have been given?

If we want…

A stationary point on the curve

If we want a curve given by $y = f(x)$ to have a stationary point at (a, b) where a and b are real numbers, we want $f'(a) = 0$ and $f(a) = b$.

An odd number of stationary points

How many stationary points can a quadratic or cubic have?

What about a degree 4 or 5 polynomial?

Local minimum or maximum

If we want a curve to have a local minimum or local maximum, what could the curve "look like"?

If $g(x)$ is a function with a local maximum at $x = 2$, can we use this information to create a function that has a local minimum at $x = 2$?

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A possible solution

	The curve is increasing for $x > 1$	Has a local minimum with y-coordinate 1	Has a point of inflection when $x = 0$
Has a stationary point at $(1,1)$	$y = x^3 + 3x^2 - 9x + 6$	$y = (x-1)^2 + 1$	$y = 3x^4 - 4x^3 + 2$
Has a local maximum when $x = -3$	$y = x^3 + 3x^2 - 9x + 6$	$y = 2x^3 + 9x^2 + 1$	$y = 1 - \frac{1}{4}x^4 - x^3$
Has an odd number of stationary points	$y = 7(x-1)^2 + 1$	$y = (x+4)^2 + 1$	$y = x^5 - x^3 + 5$