Double Trouble

Charlie has been adding fractions in the sequence $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots$ where each fraction is half the previous one:

$\frac{1}{2} + \frac{1}{4}$ $\qquad$ $\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$ $\qquad$ $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$

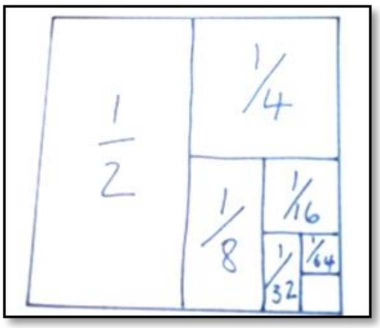
Work out the answers to Charlie's sums. What do you notice?
Will the pattern continue? How do you know?

Try writing an expression for

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots + \frac{1}{2^n}$$

Could you convince someone else that your expression is correct for all values of *n*?

Charlie drew a diagram to try to explain what was going on:



Use Charlie's diagram to explain why

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots + \frac{1}{2^n} = 1 - \frac{1}{2^n} = \frac{2^n - 1}{2^n}$$

Alison has been adding numbers in the sequence 1, 2, 4, 8 … where each number is twice the previous one:

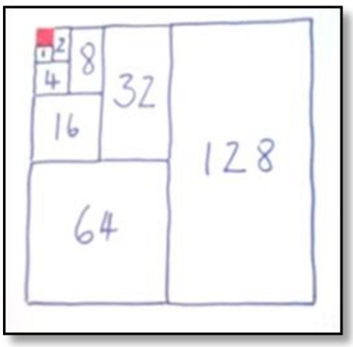
$1 + 2$ $\qquad$ $1 + 2 + 4$ $\qquad$ $1 + 2 + 4 + 8$

Work out the answers to Alison's sums. What do you notice?
Will the pattern continue? How do you know?
Try writing an expression for

$$1 + 2 + 4 + \cdots + 2^n$$

Could you convince someone else that your expression is correct for all values of *n*?

Alison drew a diagram to try to explain what was going on.



Can you use Alison's diagram to explain why

$$1 + 2 + 4 + \cdots + 2^n = 2^{n+1} - 1$$