

Charlie has been investigating square numbers. He decided to organise his work in a table:

Charlie noticed some special relationships between certain square numbers:

$$3^2 + 4^2 = 5^2 \qquad 5^2 + 12^2 = 13^2$$

Sets of integers like 3, 4, 5 and 5, 12, 13 are called **Pythagorean Triples**, because they could be the lengths of the sides of a right-angled triangle.

He wondered whether he could find any more...

1	1	3
2	4	5
3	9	7
4	16	9
5	25	11
6	36	13
7	49	15
8	64	17
9	81	19
10	100	21
11	121	23
12	144	25
13	169	27
14	196	

Handwritten notes on the table:  
 $3^2 + 4^2 = 5^2$  (written in blue next to row 4)  
 $5^2 + 12^2 = 13^2$  (written in blue next to row 12)  
 The squares 16, 25, 144, and 169 are circled in blue.  
 The numbers 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27 are written in red and connected to their corresponding squares by a bracket on the right.

Can you extend Charlie's table to find any more sets of Pythagorean Triples where the hypotenuse is 1 unit longer than one of the other sides? Do you notice any patterns? Can you make any predictions?

**Can you find a formula that generates Pythagorean Triples like Charlie's?**

**Can you prove that your formula works?**

Alison has been working on Pythagorean Triples where the hypotenuse is 2 units longer than one of the other sides.

So far, she has found these:

$$4^2 + 3^2 = 5^2 \qquad 6^2 + 8^2 = 10^2 \qquad 8^2 + 15^2 = 17^2$$

Some of these are just scaled-up versions of Charlie's triples, but some of them are new and can't be divided by a common factor (these are called **primitive triples**).

Can you find more Pythagorean Triples like Alison's?

**Can you find a formula for generating Pythagorean Triples like Alison's?**

**Can you prove that your formula works?**