The Greedy Algorithm

So far you may have looked at how the Egyptians expressed fractions as the sum of different unit fractions.

How would the Egyptians have coped with fractions with large numerators such as $\frac{115}{137}$?

Fibonacci found a strategy called the Greedy Algorithm:

At every stage, write down the largest possible unit fraction that is smaller than the fraction you're working on.

For example, let's start with $\frac{11}{12}$

The largest possible unit fraction that is smaller than $\frac{11}{12}$ is $\frac{1}{2}$

$\frac{11}{12} - \frac{1}{2} = \frac{5}{12}$, so $\frac{11}{12} = \frac{1}{2} + \frac{5}{12}$

The largest possible unit fraction that is smaller than $\frac{5}{12}$ is $\frac{1}{3}$

$\frac{5}{12} - \frac{1}{3} = \frac{1}{12}$, so $\frac{11}{12} = \frac{1}{2} + \frac{1}{3} + \frac{1}{12}$

Choose a fraction of your own and apply the Greedy Algorithm to see if you can finish up with a string of unit fractions.

Does the greedy algorithm always work?
Can all fractions be expressed as a sum of different unit fractions by applying the Greedy Algorithm?
Can you convince yourself of this?

Why do you think it is called the Greedy Algorithm? What do these words mean in a mathematical context?