

This is the start of the harmonic triangle:

$$\begin{array}{cccccccc}
 & & & & & & & \frac{1}{1} \\
 & & & & & & & & \frac{1}{2} \\
 & & & & & & & \frac{1}{2} & \\
 & & & & & & \frac{1}{3} & & \frac{1}{3} \\
 & & & & & \frac{1}{4} & & \frac{1}{12} & & \frac{1}{4} \\
 & & & \frac{1}{5} & & \frac{1}{20} & & \frac{1}{30} & & \frac{1}{20} & & \frac{1}{5} \\
 & & \frac{1}{6} & & \frac{1}{30} & & \frac{1}{60} & & \frac{1}{60} & & \frac{1}{30} & & \frac{1}{6} \\
 & & & & & & \dots & & & & & & \\
 \end{array}$$

Each fraction is equal to the sum of the two fractions below it.

Look at the triangle above and check that the rule really does work.

Can you work out the next two rows?

The n th row starts with the fraction $\frac{1}{n}$, so the first diagonal goes:

$$\frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots$$

Take a look at the second diagonal:

$$\frac{1}{2}, \frac{1}{6}, \frac{1}{12}, \frac{1}{20}, \dots$$

Can you see a pattern?

What fraction will appear in the second position on the n th row?

Can you prove it?

What about the third and fourth diagonals?