

The ancient Egyptians didn't write fractions with a numerator greater than 1 - they wouldn't, for example, write $\frac{2}{7}$, $\frac{5}{9}$, $\frac{123}{467}$, ...

Instead they wrote fractions like these as a sum of **different** unit fractions.

There are several NRICH problems based on Egyptian fractions. The first, Keep it Simple (<http://nrich.maths.org/6540>), explores unit fractions.

In this problem we are going to start by considering how the Egyptians might have written fractions with a numerator of 2 (i.e. of the form $\frac{2}{n}$).

For example,

$$\frac{2}{3} = \frac{1}{3} + \frac{1}{3} \text{ (but since these are the same, this wasn't allowed), or}$$

$$\frac{2}{3} = \frac{1}{3} + \frac{1}{4} + \frac{1}{12}, \text{ or}$$

$$\frac{2}{3} = \frac{1}{3} + \frac{1}{5} + \frac{1}{20} + \frac{1}{12}, \text{ or}$$

$$\frac{2}{3} = \frac{1}{3} + \frac{1}{6} + \frac{1}{30} + \frac{1}{20} + \frac{1}{12}, \text{ or}$$

$$\frac{2}{3} = \frac{1}{4} + \frac{1}{12} + \frac{1}{7} + \frac{1}{42} + \frac{1}{31} + \frac{1}{930} + \frac{1}{21} + \frac{1}{420} + \frac{1}{13} + \frac{1}{156}$$

and so on, and so on!!

You might want to check that these are correct.

BUT wouldn't it be simpler to write it as the sum of just two different unit fractions?

For $\frac{2}{3}$ that's quite easy, $\frac{2}{3} = \frac{1}{2} + \frac{1}{6}$

But is it always so easy? Try some other fractions with a numerator of 2. Can they also be written as the sum of just two different unit fractions?

Can all fractions with a numerator of 2 (i.e. of the form $\frac{2}{n}$) be written as the sum of just two different unit fractions?

Can you find an efficient method for doing this?

Next, you might want to explore fractions of the form $\frac{3}{n}$, $\frac{4}{n}$, $\frac{5}{n}$...