Alison has been exploring sums with surds. She used a spreadsheet to make columns for square roots, and then added together various combinations.

Here is one of the sums she worked out:

$$\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \dots + \frac{1}{\sqrt{99}+\sqrt{100}}$$

The answer surprised her!
Can you find a way to evaluate the sum without using a calculator or spreadsheet?
If you would like a hint, look at the bottom of the page.

Can you find other similar sums with surds that give whole number answers?

Hint
When a fraction contains surds, we often choose to multiply the numerator and denominator by an expression that gets rid of any surds in the denominator.

Knowing that $(a + b)(a - b) = a^2 - b^2$ might help.