Farey Sequences

If I gave you a list of decimals, you might find it quite straightforward to put them in order of size. But what about ordering fractions?

A man called John Farey investigated sequences of fractions in order of size - they are called Farey Sequences.

The third Farey Sequence, F_3, looks like this:

$$\frac{0}{1} \quad \frac{1}{3} \quad \frac{1}{2} \quad \frac{2}{3} \quad \frac{1}{1}$$

It lists in order all the fractions between 0 and 1, in their simplest forms, with denominators up to and including 3.

Here is F_4:

$$\frac{0}{1} \quad \frac{1}{4} \quad \frac{1}{3} \quad \frac{1}{2} \quad \frac{2}{3} \quad \frac{3}{4} \quad \frac{1}{1}$$

Write down F_5.

Which extra fractions are in F_5 which weren't in F_4?
Which extra fractions will be in F_6 that weren't in F_5?
Where will they appear in the sequence?

There are lots of questions you could explore about Farey Sequences. Here are just a few that we thought of:

- How many extra fractions are there in F_{11} that aren't in F_{10}?
- How many extra fractions are there in F_{12} that aren't in F_{11}?
- Is every Farey Sequence longer than the one before? How do you know?
- Is there a way of working out how many fractions there will be in the next sequence?
- So far, all the Farey Sequences except F_1 have contained an odd number of fractions. Can you find a Farey Sequence with an even number of fractions?