Forwards add Backwards Assignment (Nrich Live Problem)

- Vivaan BV

1. The number can be made by adding a -digit number with its reversal:
   For example,621 + 126. Can you find the other two ways of making a total of 747 in this way?
   Ans:
   1. 255 + 552 = 747
   2. 423 + 324 = 747

2. Can you find all the totals between 700 and 800 that can be made from a number plus its reversal? (There are more than five...)
   Ans:
   1. 706 Family:
      a. 706 = 551 + 155
      b. 726 = 561 + 165
      c. 746 = 571 + 175
      d. 766 = 581 + 185
      e. 786 = 591 + 195

   2. 707 Family:
      a. 707 = 601 + 106
      b. 727 = 611 + 116
      c. 747 = 621 + 126
      d. 767 = 631 + 136
      e. 787 = 641 + 146

3. Can you explain how you know you have found all the possible totals?
   Ans: I know that I have found all the possible totals because
   - The digit that is in tens place of the final 3 digit number(i.e.the middle digit of the 3 digit number) should be even since this digit when divided by 2, gives the middle digit of the number and its reversal which will be added to get the required number.
   - This tens place digit should be less than 4.
   - So ,in each of the 2 families, only 5 numbers are only possible. i.e. the center digit can be only 0, 2, 4, 6, 8.

- Now, the first digit of the required number should be 7 as the numbers should be in between 700 and 800.

4. How many totals between 300 and 400 can be made from a number plus its reversal? And between 800 and 900...?
   Ans:
   1. 300 to 400
      a. 302 Family:
         i. 302 = 151 + 151
         ii. 322 = 161 + 161
         iii. 342 = 171 + 171
         iv. 362 = 181 + 181
         v. 382 = 191 + 191
      b. 303 Family:
         i. 303 = 201 + 102
         ii. 323 = 211 + 122
         iii. 343 = 221 + 122
         iv. 363 = 231 + 132
         v. 383 = 241 + 142

   2. 800 to 900
      a. 807 Family:
         i. 807 = 156 + 651, 255 + 552, 354 + 453
         ii. 827 = 166 + 661, 265 + 562, 364 + 463
         iii. 847 = 176 + 671, 275 + 572, 374 + 473
         iv. 867 = 186 + 681, 285 + 582, 384 + 473
         v. 887 = 196 + 691, 295 + 592, 394 + 493
      b. 808 Family:
         i. 808 = 107 + 701, 206 + 602, 305 + 503, 404 + 404
         ii. 828 = 117 + 711, 216 + 612, 315 + 513, 414 + 414
         iii. 848 = 127 + 721, 226 + 622, 325 + 523, 424 + 424
         iv. 868 = 137 + 731, 236 + 632, 335 + 533, 434 + 434
         v. 888 = 147 + 741, 246 + 642, 345 + 543, 444 + 444

5. Can you work out how to make a total of 1251 by adding a 3-digit number to its reversal?
   Ans: 972 + 279

I got this answer by:

- 1 is in the units place of 1251.
- So, we should get 11 by adding the digits at the units place as 1 is not possible.
- Considering 11, 1 will be carried to tens place.
- So, the number in the tens place should be the same in both the 3 digit numbers and the sum should be 14 as 1 is already carried from the units place as 10.
- 7 + 7 = 14 and the digits are also the same. So, 7 + 7 = 14 + 1 = 15.
- Therefore, 15 should come in the tens place in the answer (1251), which has come.
- So we should carry 1 from 15 to hundreds place.
- Since we already have the units place numbers (2 and 9) we should just write them in the reverse order (9 and 2).
- 9 + 2 = 11 + 1 = 12
- So, in conclusion, 972 + 279 = 1251.

6. Can you find all the totals between 1200 and 1300 that can be made from a number plus its reversal? (There are more than five...)

Ans:

1. 1211 Family:
    a. 1211 = 358 + 853, 457 + 754, 556 + 655, 952 + 259
    b. 1231 = 368 + 863, 467 + 764, 566 + 665, 962 + 269
    c. 1251 = 378 + 873, 477 + 774, 576 + 675, 972 + 279
    d. 1271 = 388 + 883, 487 + 784, 586 + 685, 982 + 289
    e. 1291 = 398 + 893, 497 + 794, 596 + 695, 992 + 299
    f. 1200 = 1200 + 0021

2. 1212 Family:
    a. 1212 = 606 + 606
    b. 1232 = 616 + 616
    c. 1252 = 626 + 626
    d. 1272 = 636 + 636
    e. 1292 = 646 + 646

7. And what about totals between 1900 and 2000...?

Ans:

1. 1918 Family:
    a. 1918 = 959 + 959
    b. 1938 = 969 + 969

c. 1958 = 979 + 979
d. 1978 = 989 + 989
e. 1998 = 999 + 999

8. Are there any numbers between 200 and 300 that can be made by subtracting a three digit number from its reversal? (There are fewer than five...)
   Ans: = 297 = 603 - 306, 502 - 205

9. Are there any numbers between 1200 and 1300 that can be made by subtracting a four digit number from its reversal? five, but more than one...)
   Ans:
   1. 1269 = 1300 - 0031
   2. 1278 = 3081 - 1803

10. Have you found all the possibilities? Can you explain how you know?
    Ans: Yes, I have found all the possibilities. There are 2 possibilities = 1269 and 1278.
    - The first answer:
      1. There is always at least 1 way for subtraction of a 3 or 4 digit number and its reversal to get the answer.
      2. For getting this number, we must take the higher number given in the range (here, take 1300 in the range: 1200 and 1300)
      3. Subtract the higher number from its reversal. U get your first answer.(here 1300-0031 - 1269.)
    - Now to find the second answer. Here is my logic:
      1. Answers should be between 1200 and 1300.
      2. If you subtract a number and its reversed number, this answer is always multiple of 9. So our answer should be multiple of 9.
      3. If we check 1269 - this is multiple of 9.
      4. Now, let us consider the first number as A and its reversal as Z.
      5. Now, since the first digit of answer is 1, the difference of first digits of A and Z should be 1. So if we take the first digits of A as 2 and first digit of Z as 1, automatically the last digit of A becomes 1 and Z becomes 2.
      6. So, the first digit of answer is 1, and last digit of answer is 9.
      7. Now, the second digit of answer is 2, so we have 3 digits of our answer 1 2 ___ 9. If this number should be multiple of 9, the missing digit should be 6.

8. Again, if we take the difference of the first digit of A and Z as 2 (assuming we carry a number to subtract bigger from smaller. Then we have the first digit of A as 3 and Z as 1 and the last digit of A as 1 and the last digit of Z as 3.
9. Now if we have 1 and 3 in the unit place, the units place of difference is 8.
10. We have 3 digits of our answer - 1 2 ___ 8. So to make this multiple of 9, the 3rd digit is 7.
11. Since our answer should lie between 1200 and 1300, these are only 2 options.
12. **So, the answers are 1269 and 1278.**