Forwards Add Backwards: Complete Solutions

Full worked answers and reasoning for the NRICH investigation.

1. Other Ways of Making 747

A 3-digit number abc added to its reverse cba gives:
$(100a + 10b + c) + (100c + 10b + a) = 101(a + c) + 20b$.
For no carrying, $a + c = 7$ and $2b = 4$ (so $b = 2$).

- Given: $621 + 126 = 747$
- **Second way:** $522 + 225 = 747$
- **Third way:** $423 + 324 = 747$

2. All Totals Between 700 and 800 (10 Totals)

Any sum between 700 and 800 must have hundreds digit 7. This can happen in two ways:

- **Case A (No carry to hundreds):** $a + c = 7$.
  The tens sum is 2b. Since b ranges from 0 to 4 (so no carry):
  • $b = 0 \rightarrow$ **707** (e.g. 700 + 007, or non-zero: 601 + 106)
  • $b = 1 \rightarrow$ **727** (e.g. 611 + 116)
  • $b = 2 \rightarrow$ **747** (e.g. 621 + 126)
  • $b = 3 \rightarrow$ **767** (e.g. 631 + 136)
  • $b = 4 \rightarrow$ **787** (e.g. 641 + 146)
- **Case B (With carry to hundreds):** $a + c = 6$, plus 1 carried from the tens column.
  The tens sum 2b must be ≥ 10, so b ranges from 5 to 9:
  • $b = 5$ (2b = 10, carry 1, unit 6) → **716** (e.g. 551 + 155)
  • $b = 6$ (2b = 12, carry 1, unit 6) → **736** (e.g. 561 + 165)
  • $b = 7$ (2b = 14, carry 1, unit 6) → **756** (e.g. 571 + 175)
  • $b = 8$ (2b = 16, carry 1, unit 6) → **776** (e.g. 581 + 185)
  • $b = 9$ (2b = 18, carry 1, unit 6) → **796** (e.g. 591 + 195)

Complete list of 10 totals: 707, 716, 727, 736, 747, 756, 767, 776, 787, 796.

3. Totals Between 300–400 and 800–900

- **Between 300 and 400:** 10 totals (303, 312, 323, 332, 343, 352, 363, 372, 383, 392).
- **Between 800 and 900:** 10 totals (808, 817, 828, 837, 848, 857, 868, 877, 888, 897).

4. 4-Digit Totals (1200–1300 and 1251)

To get a 4-digit sum around 1200, $a + c = 11$ (since units digit must be 1 and sum exceeds 1000).
• Units column: 1 (with 1 carried).
• Tens column: $2b + 1$.
• Hundreds/Thousands column: $11 + \text{carry}$ from tens.
For **1251**: tens digit is 5, so $2b + 1 = 5 \rightarrow 2b = 4 \rightarrow b = 2$ (no carry into hundreds).
With $a + c = 11$ and $b = 2$: e.g., **922 + 229 = 1251**, **823 + 328 = 1251**, **724 + 427 = 1251**, **625 + 526 = 1251**.
• **All totals between 1200 and 1300:** 10 totals (1211, 1231, 1251, 1271, 1291 without tens carry; and 1201, 1221, 1241, 1261, 1281 from $a + c = 12$ with carry).
• **Between 1900 and 2000: 0 totals**. The maximum possible sum of two 3-digit numbers is $999 + 999 = 1998$, but for reversals the max is $998 + 899 = 1897$. None fall in 1900–2000.

5. Subtraction Extension

Reversal subtraction for a 3-digit number is: $(100a + 10b + c) - (100c + 10b + a) = 99(a - c)$.
Every 3-digit subtraction result **must be a multiple of 99**.
• **Between 200 and 300:** The only multiple of 99 in this range is $99 \times 3 = 297$ (e.g. 542 − 245 = 297). There is exactly **1 number**.
• **4-digit subtraction between 1200 and 1300:** For 4-digit numbers $abcd - dcba = 999(a - d) + 90(b - c)$.
The only values between 1200 and 1300 are **1269** and **1278** (fewer than 5, more than 1).