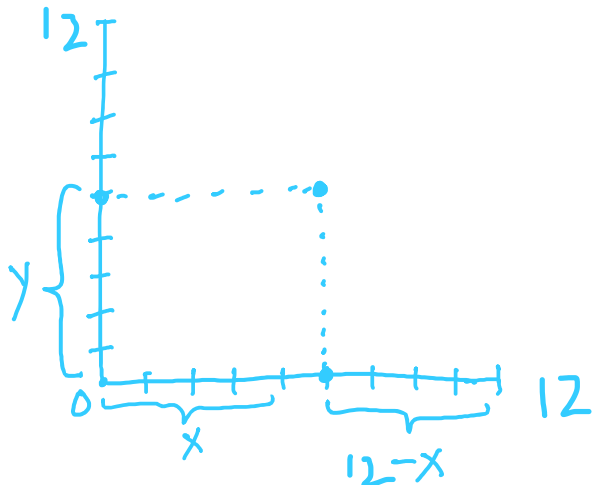


NRICH 3D and 2D Treasure Finding Solution:

Level 1:

After some experimenting, we found an optimal strategy for the 12×12 2D Treasure Grid. We know the values of x and y are non-negative (≥ 0) from $1 - 12$. If we input $(0, 0)$, we know that $x + y$ must sum to the number of steps the algorithm returns.



Now we can input $(12, 0)$. * In the graph above, we can now find $(12 - x) + y$. If we know $(12 - x) + y$ and $x + y$, we can set up a simultaneous equation and solve for x . After we acquire the value of x , we can substitute it into the first equation and find the value of y . Therefore, the least number of guesses we need to find the treasure in any case scenario is 3 guesses.

Let's use a real example from NRICH to demonstrate the optimal strategy. First, let's input $(0, 0)$:

$(0,0)$ is 15 steps away

Now let's input the maximum possible value of x , which is 12:

$(9,0)$ is 7 steps away

Now we can deduce the value of x . We know that x is closer to 0 than to 9. But for the actual value of x , we can see with that $x + y = 15$, while $(9 - x) + y = 7$. We can solve the equation to get $x = 5$.

Now we know the value of x , we can resubstitute it into $x + y = 15$. Therefore $y = 10$. If we input $x = 5$ and $y = 10$, we can see that...

You have found the treasure!

Level 2:

For this method, use the exact same method from level 1 (see above). Because the algorithm returns the positive value of the difference of x and sum of the positive value of the difference of y , we can simply replace $(0, 0)$ into the bottom left coordinates of the graph. Then afterwards, instead of $(9, 0)$ we input the bottom right value of the graph and solve for x and y just like the previous example using our simultaneous equations: $x + y$ and $(N - x) + y$. Note that N is the value of x you guess in your second guess, in this case the bottom right of the graph. No example will be provided. Try it out yourself!

Level 3:

For this level, we are only allowed one guess outside the purple region. The purple regions are either horizontal, vertical or diagonal. The optimal solutions below are based off the same method used in level 1. Note that every time you refresh the level, the purple region and the coordinates of the graph nearly always change.

Horizontal:

In this case, the optimal solution requires 4 guesses. The first guess will be to guess any y coordinates (only if it's within the purple box) and the minimum value of x , and the second guess will be the same y coordinates with the maximum value of x . Now using simultaneous equations, you can determine the exact value of x . However, you don't know the direction y is going (unless one of the possible values of y goes off the grid), because you aren't at the bottom of the grid, in which y can only go up. So, in the third guess, change y by 1 in any direction. If the number of steps increases, your y value must be the other way. Therefore, in the fourth guess with the direction known you can secure the value of x and y .

Vertical:

In this case, the optimal solution also requires 4 guesses. The first guess will be to guess any x coordinates (only if it's within the purple box) and the minimum value of y , and the second guess will be the same x coordinates with the maximum value of y . Now using simultaneous equations, you can determine the exact value of y . However, you don't know the direction x is going (unless one of the possible values of x goes off the grid), because you aren't at the

left side of the grid, in which the value of x can only increase. So, in the third guess, change x by 1 in any direction. If the number of steps increases, your x value must be the other way. Therefore, in the fourth guess with the direction known you can secure the value of x and y .

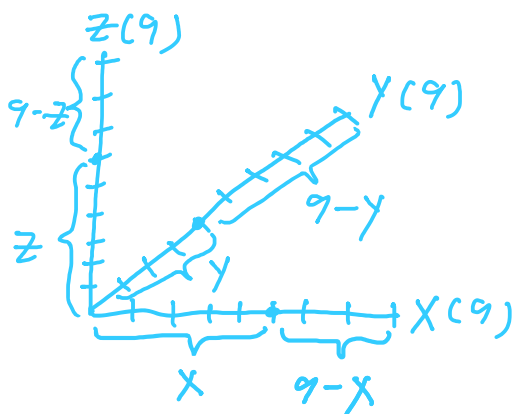
Diagonal:

The least number of guesses you need to secure the coordinates of the treasure is also 4. Let's assume the diagonal begins at bottom left and ends at top right. In this case, you first enter the bottom left coordinates. If it's not an even number, add by 1. Then increase your first coordinates x and y value by half of the number of steps (even) as your second guess. This ensures your coordinates stays within the purple line, in the exact centre parts of the line. Now you'll notice that after calculations or trial and error, you have two sets of coordinates that can be the possible answer. For the third guess, change the x or y value by 1 or -1 (not both), so you can find the general direction of the treasure's coordinates (If the number of steps increases, you'll know the value must go the other way). Then, enter the set of coordinates that is in the general direction found in the third guess.

Finally, for all three methods, check back to the level 1 method to create simultaneous equations to solve for the values of x and y . They take one extra step than the original method because you need to find the general direction before being able to find the final coordinates.

Level 4:

At first, visualising a 3D graph with complicated coordinates might stress you. But after a while of experimenting, you can easily find a pattern. We can see that the values of x , y and z are non-negative and have random values from 1 – 9. If we input $(0, 0, 0)$, because x , y and z can only be bigger or equal to 0, we know that $x + y + z$ equals the number of squares from the treasure.



Now let's find the value of x . If we input the maximum value of x into the graph, $(9, 0, 0)$ *, then we can find the value of $(9 - x) + y + z$. Now we easily set up a simultaneous equation and solve for x .

Afterwards, we can use the same method to solve for y . We input $(0, 9, 0)$ * and find $x + (9 - y) + z$. After we acquire the values of x and y , we substitute the values into the first equation and solve for z .

Overall, the whole process only took 4 guesses. 4 is the least number of guesses you need to find any set of coordinates. And that's the optimal solution.

Now, let's use a real example from NRICH:

First, we input $(0, 0, 0)$:

$(0, 0, 0)$ is 14 steps away

Now, let's input $(9, 0, 0)$:

$(9, 0, 0)$ is 17 steps away

We can see that $x + y + z = 14$, and $(9 - x) + y + z = 17$. Subtracting the first equation from the second equation, we get $2x = 6$, therefore $x = 3$. Now let's input $(0, 9, 0)$ to find the value of y :

$(0, 9, 0)$ is 13 steps away

We know that $x + y + z = 14$, and $x + (9 - y) + z = 13$. Therefore after solving the simultaneous equation, we get $y = 5$. Now substituting the values of x and y into $x + y + z = 14$, we get $z = 6$. If we input $(3, 5, 6)$, we get...

You have found the treasure!

Overall, for any grid, 3D or 2D, even if the grid is not square, you can still use the methods above. Just adapt by changing some of the variables based off the question (Eg. The side length values). Thank you for reading our proof.

*Why did we always use nines and zeros? Mainly because it's simpler to calculate. Let's assume the treasure is at (7, 2). If we input (0, 0), the treasure would be 9 steps away ($x + y$). Then if we then input another number like 8 as the x value, we would get the treasure is 3 steps away ($(N - x) + y$) (N is the number you input as the x value). However, we can still find the exact value of x using our simultaneous equations: $(8 - x) + y = 3$, and $x + y = 9$. Overall, you can use whichever number you please. You can even start with (12, 12), then use (0, 12) to find the treasure within 3 guesses. Note that this is based off level 1, and the theory can be applied to any other level.