

Let us call our mystery number(s) xyz , where variables represents a digit in place value. Therefore, xyz can be accurately represented as

hundreds	tens	ones
x	y	z
$100 \cdot x$	$10 \cdot y$	$1 \cdot z$

We assume that it must be a three-digit number originally/reversed because the maximum sum of two two-digit numbers is 198, and four digits are already greater than 800.

Therefore, xyz represents a number with value $100x + 10y + z$. Likewise, the reversed number, zyx , has a number with value $100z + 10y + x$.

That means that the sum of xyz and its reverse (zyx) is:
 $(100x + 10y + z) + (100z + 10y + x)$
 $= 101x + 20y + 101z$ which can be factorised and rearranged to
 $= 101(x + z) + 20y \dots \textcircled{1}$

In the question, we're told the sum of xyz and zyx is greater than 700 but less than 800. This is useful to us because it helps us figure out that the sum of the hundreds digits must be 7 if the sum of the tens digits is less than 10, and the sum of the hundreds digits must be 6 if the tens digits is 10 or more in total. To put it algebraically:

(all digits are integers, of course)

When $y < 5$, $x + z = 7$

Substitute $x + z = 7$ into $\textcircled{1}$:

When $y = 0$: $101(7) + 20(0) = 707$
 When $y = 1$: $101(7) + 20(1) = 727$
 When $y = 2$: $101(7) + 20(2) = 747$
 When $y = 3$: $101(7) + 20(3) = 767$
 When $y = 4$: $101(7) + 20(4) = 787$

Clearly, if y got larger than this, the sum would be greater than 800, breaking the condition.

When $y \geq 5$, $x + z = 6$

Substitute $x + z = 6$ into $\textcircled{1}$:

When $y = 5$: $101(6) + 20(5) = 706$
 When $y = 6$: $101(6) + 20(6) = 726$
 When $y = 7$: $101(6) + 20(7) = 746$
 When $y = 8$: $101(6) + 20(8) = 766$
 When $y = 9$: $101(6) + 20(9) = 786$

If y increased any further, which it cannot, since it must be a single-digit positive/neutral integer, the total would be more than 800. If y decreased any further, we would break the 'greater than 700' condition, which we didn't.

We also know that $x + z \neq 5$ because the maximum that can be gotten from $20y$ is 180 (as $0 \leq y \leq 9$) and 680 is too low from 700. $x + z = 8$ already breaks the condition as y is positive, and $x + z = 8$ is greater than 10.

Why does it have to be $y < 5$? digits: 4, 3, 2, 1 or 0

We can take a look at $\textcircled{1}$, substituting $x + z = 7$:
 $101(7) + 20y$
 We know that it has to be less than 800:

$$707 + 20y < 800$$

$$20y < 93$$

$$y < 4.65$$

So the only positive/neutral integers that satisfy this condition is: 4, 3, 2, 1 and 0

Why does it have to be $y \geq 5$? digits: 5, 6, 7, 8, 9

We can take a look at $\textcircled{1}$, substituting $x + z = 6$:

$101(6) + 20y$
 We know that it has to be greater than 700:

$$606 + 20y > 700$$

$$20y > 94$$

$$y > 4.7$$

We also know that a single digit cannot get any greater than 9, so the sum of the two numbers won't be greater than 800. So the only positive integers that satisfy these conditions are:

5, 6, 7, 8 and 9

In conclusion, we can tell that all the totals between 700 and 800 that can be made from a number plus its reversal are:

707 727 747 767 787 (palindromes)

706 726 746^{and} 766 786 (not
palindromes).

And my full solution above states how I derived my answer.