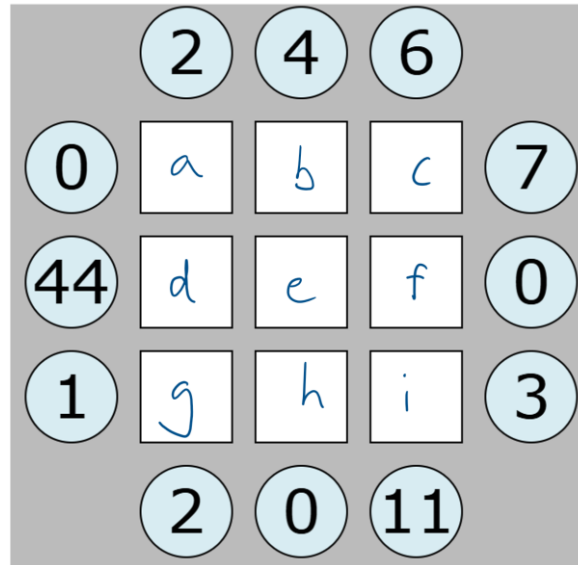


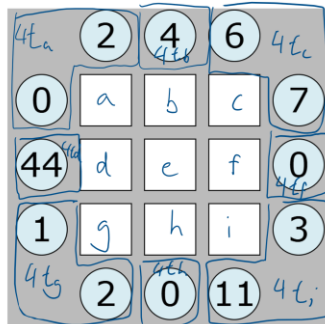
Simplifying the problem:



My approach to solve this problem is to find the general algebraic solution for every 3x3 connecting average puzzle, hence we could be able to find the solution for every single set of values around the grid.

Firstly, to simplify this: we can use this notation:

$$t_n = (\text{total of the adjacent squares which have a known value}) \div 4$$



Therefore, for the term (a):

$$a = \frac{b}{4} + \frac{d}{4} + t_a$$

(This is the average which is required to find a)

Which can be arranged as:

$$a - \frac{b}{4} - \frac{d}{4} = t_a$$

Which therefore can be presented as this:

$$a - \frac{b}{4} + 0c - \frac{d}{4} + 0e + 0f + 0g + 0h + 0i = t_a$$

Hence can be presented as this:

$$\begin{aligned} a - \frac{b}{4} - \frac{c}{4} + 0d + 0e + 0f + 0g + 0h + 0 \\ = t_a + 0t_b + 0t_c + 0t_d + 0t_e + 0t_f + 0t_g + 0t_h + 0t_i \end{aligned}$$

Which can be represented as a matrix of each coefficient:

$$\left(1 - \frac{1}{4} \ 0 - \frac{1}{4} \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \right)$$

Which therefore, for every single grid square from a to i, if we rearrange all of the values, we get this:

$$\left[\begin{array}{cccccccccc|cccccccccc} 1 & -\frac{1}{4} & 0 & -\frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{4} & 1 & -\frac{1}{4} & 0 & -\frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{4} & 1 & 0 & 0 & -\frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{4} & 0 & 0 & 1 & -\frac{1}{4} & 0 & -\frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{4} & 0 & -\frac{1}{4} & 1 & -\frac{1}{4} & 0 & -\frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{4} & 0 & -\frac{1}{4} & 1 & 0 & 0 & -\frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{4} & 0 & 0 & 1 & -\frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\frac{1}{4} & 0 & -\frac{1}{4} & 1 & -\frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{4} & 0 & -\frac{1}{4} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} \right]$$

Where in the right we see an identity matrix, and if we perform Gaussian elimination, and do operations on each row to convert the left 9x9 matrix into an identity matrix, we convert the identity matrix in the right into the inverse of the left matrix. So once we find the inverse:

We want to convert the left matrix into the identity matrix so we can be able to find the equations of t_n of which we could use to find the value of each unfilled grid square.

Here is the inverse of the matrix:

$$\begin{pmatrix} \frac{67}{56} & \frac{11}{28} & \frac{1}{8} & \frac{11}{28} & \frac{1}{4} & \frac{3}{28} & \frac{1}{8} & \frac{3}{28} & \frac{3}{56} \\ \frac{11}{28} & \frac{37}{28} & \frac{11}{28} & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & \frac{3}{28} & \frac{5}{28} & \frac{3}{28} \\ \frac{1}{8} & \frac{11}{28} & \frac{67}{56} & \frac{3}{28} & \frac{1}{4} & \frac{11}{28} & \frac{3}{56} & \frac{3}{28} & \frac{1}{8} \\ \frac{11}{28} & \frac{1}{4} & \frac{3}{28} & \frac{37}{28} & \frac{1}{2} & \frac{5}{28} & \frac{11}{28} & \frac{1}{4} & \frac{3}{28} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & \frac{1}{2} & \frac{3}{2} & \frac{1}{2} & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{3}{28} & \frac{1}{4} & \frac{11}{28} & \frac{5}{28} & \frac{1}{2} & \frac{37}{28} & \frac{3}{28} & \frac{1}{4} & \frac{11}{28} \\ \frac{1}{8} & \frac{3}{28} & \frac{3}{56} & \frac{11}{28} & \frac{1}{4} & \frac{3}{28} & \frac{67}{56} & \frac{11}{28} & \frac{1}{8} \\ \frac{3}{28} & \frac{5}{28} & \frac{3}{28} & \frac{1}{4} & \frac{1}{2} & \frac{1}{4} & \frac{11}{28} & \frac{37}{28} & \frac{11}{28} \\ \frac{3}{56} & \frac{3}{28} & \frac{1}{8} & \frac{3}{28} & \frac{1}{4} & \frac{11}{28} & \frac{1}{8} & \frac{11}{28} & \frac{67}{56} \end{pmatrix}$$

Which is the coefficients of each term of t_n .

So in the end,

$$\begin{aligned} a &= \frac{67t_a}{56} + \frac{11t_b}{28} + \frac{1t_c}{8} + \frac{11t_d}{28} + \frac{1t_e}{4} + \frac{3t_f}{28} + \frac{1t_g}{8} + \frac{3t_h}{28} + \frac{3t_i}{56} \\ b &= \frac{11t_a}{28} + \frac{37t_b}{28} + \frac{11t_c}{28} + \frac{1t_d}{4} + \frac{1t_e}{2} + \frac{1t_f}{4} + \frac{3t_g}{28} + \frac{5t_h}{28} + \frac{3t_i}{28} \\ &\vdots \end{aligned}$$

In the end, we can generalise all the equations to corner, edge and centre as every grid square is just the rotational/reflected symmetry of the corner, edge or centre.

Here is the generalised formula for each outcome:

Corner:

Corner:

$$a = \frac{67t_a}{56} + \frac{11t_b}{28} + \frac{1t_c}{8} + \frac{11t_d}{28} + \frac{1t_e}{4} + \frac{3t_f}{28} + \frac{1t_g}{8} + \frac{3t_h}{28} + \frac{3t_i}{56}$$

Edge:

$$b = \frac{11t_a}{28} + \frac{37t_b}{28} + \frac{11t_c}{28} + \frac{1t_d}{4} + \frac{1t_e}{2} + \frac{1t_f}{4} + \frac{3t_g}{28} + \frac{5t_h}{28} + \frac{3t_i}{28}$$

Centre:

$$e = \frac{t_a}{4} + \frac{t_b}{2} + \frac{t_c}{4} + \frac{t_d}{2} + \frac{3t_e}{2} + \frac{t_f}{2} + \frac{t_g}{4} + \frac{t_h}{2} + \frac{t_i}{4}$$

Summary:

This is the generalised formula for every grid square involved in the 3x3 connecting squares puzzle, and this method could be used for every other puzzle type, as this is just a direct way of solving a generated simultaneous equation that is rooted from the 3 by 3. An issue that might arise is that high degree matrices might be hard to calculate by hand, but this method works well with low degree ones.

However If you would like to use an even more direct approach, you can directly do Gauss-Jordan elimination to get row echlon form in the matrix calculator by itself:

$$\left(\begin{array}{cccccccc|c} 1 & -\frac{1}{4} & 0 & -\frac{1}{4} & 0 & 0 & 0 & 0 & 0 & t_a \\ -\frac{1}{4} & 1 & -\frac{1}{4} & 0 & -\frac{1}{4} & 0 & 0 & 0 & 0 & t_b \\ 0 & -\frac{1}{4} & 1 & 0 & 0 & -\frac{1}{4} & 0 & 0 & 0 & t_c \\ -\frac{1}{4} & 0 & 0 & 1 & -\frac{1}{4} & 0 & -\frac{1}{4} & 0 & 0 & t_d \\ 0 & -\frac{1}{4} & 0 & -\frac{1}{4} & 1 & -\frac{1}{4} & 0 & -\frac{1}{4} & 0 & t_e \\ 0 & 0 & -\frac{1}{4} & 0 & -\frac{1}{4} & 1 & 0 & 0 & -\frac{1}{4} & t_f \\ 0 & 0 & 0 & -\frac{1}{4} & 0 & 0 & 1 & -\frac{1}{4} & 0 & t_g \\ 0 & 0 & 0 & 0 & -\frac{1}{4} & 0 & -\frac{1}{4} & 1 & -\frac{1}{4} & t_h \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{4} & 0 & -\frac{1}{4} & 1 & t_i \end{array} \right)$$

Also note that t(e) is always just 0 because all adjacent squares are unknown values.

Sources:

Wolfram alpha to calculate the 9x9 matrix: [Wolfram alpha 9x9 matrix calculation](#)