

NRICH Live Problem: More Dicy Operations.

Let's first see the problem.

The computer generates 3 numbers, in which we have to use them to create a two digit and one digit number. So they multiply to get as close as possible to 100. Our goal is to find the optimum answer in one or two goes. I have two methods. But let's solve all the questions before we get on to the optimum methods.

How close can you get to the target (100)?

Obviously, we can get to the target (100) as long as we're lucky. If we keep on resetting / jumbling the numbers, we will eventually end up with digits that can form two numbers (two digit and one digit) that can multiply to *exactly* 100, like {50 and 2}, {25 and 4} and {20 and 5}. But the chances of finding these digits are extremely unlikely, each one only having a chance of 0.6%*, having a combined chance of 1.8% to find any one of the sets.

**This is because it can appear in 6 different form, (Eg, 123, 132, 213, 232, 312, 321). Therefore it has a $6/10^3$ chance of finding.*

How are you deciding where to put the digits?

(Scroll down to see the second method, which includes a detailed explanation for this question)

First method:

Obviously, we could try trial and error, which I disapprove of. There are at most 3! Methods for guessing the order for three different digits (Eg. 3,4,5). Which is equal to: $3*2*1 = 6$ possible answers. So our probability of guessing the correct answer in the first two goes is $2/6 = 1/3$.

But if there are two of the same numbers (Eg. 4,4,6) then there would be $3!/2 = 3$ possible ways to guess the answer. So the chance of guessing the answer in the first two goes. is $2/3$.

But if there are *three* of the same numbers (Eg, 5,5,5) then obviously there will only be one formation available. Even though the chances for this to occur is *extremely* low, I would still like to cover it (a 1 in 1000 chance).

Second method:

This method is a method I developed myself to find the optimum combination, which requires some thinking before typing in the digits. Let's assume the digits are x, y and z. My method is that we divide 100 (The dividend) by one of those digits (acting as a divisor), and try to use the other digits to get as close as possible to the quotient (**Dividend / Divisor = Quotient**), and then measure the difference from 100. Let's see an example:

(The computer generated 2, 4 and 7, so we'll be using these numbers as examples)

So now we will be trying to use the digits 2, 4 and 7 to form a two digit number and a one digit number so that they can multiply to get as close as possible to 100.

Let's use 2 as the divisor first. $100/2 = 50$. The closest 4 and 7 can form to 50 is 47. Now, the actual difference from 100 it will have is $(50 - 47)*2 = 6$.

Now let's try 7 as the divisor. $100/7 = 14.286$ (to 3 decimal places). The closest 2 and 4 can form to 14* (rounded) is 24. Now sometimes we don't have to calculate the exact difference the answer has to 100, just use common sense. It's obvious this answer will be much further away from 100 than the last answer ($14*24$ and $2*47$, you can tell which one is closer to 100). So we remove this possibility.

Finally, let's try 4 as the divisor. $100/4 = 25$. The closest 2 and 7 can form to 25 is 27. So the actual difference from 100 will be $(27 - 25)*4 = 8$.

So now it's obvious that 2 and 47 will multiply to form the number closest to 100. But to examine our method, we can find that $14*24=336$, $336-100=236$; $4*27=108$, $108-100=8$; and finally, $2*47=94$, $100-94=6$. $6<8<236$, so $2*47$ is the optimum answer.

** always remove the decimals as they just make the calculation messy and difficult. It won't change the answer by a huge amount anyways.*