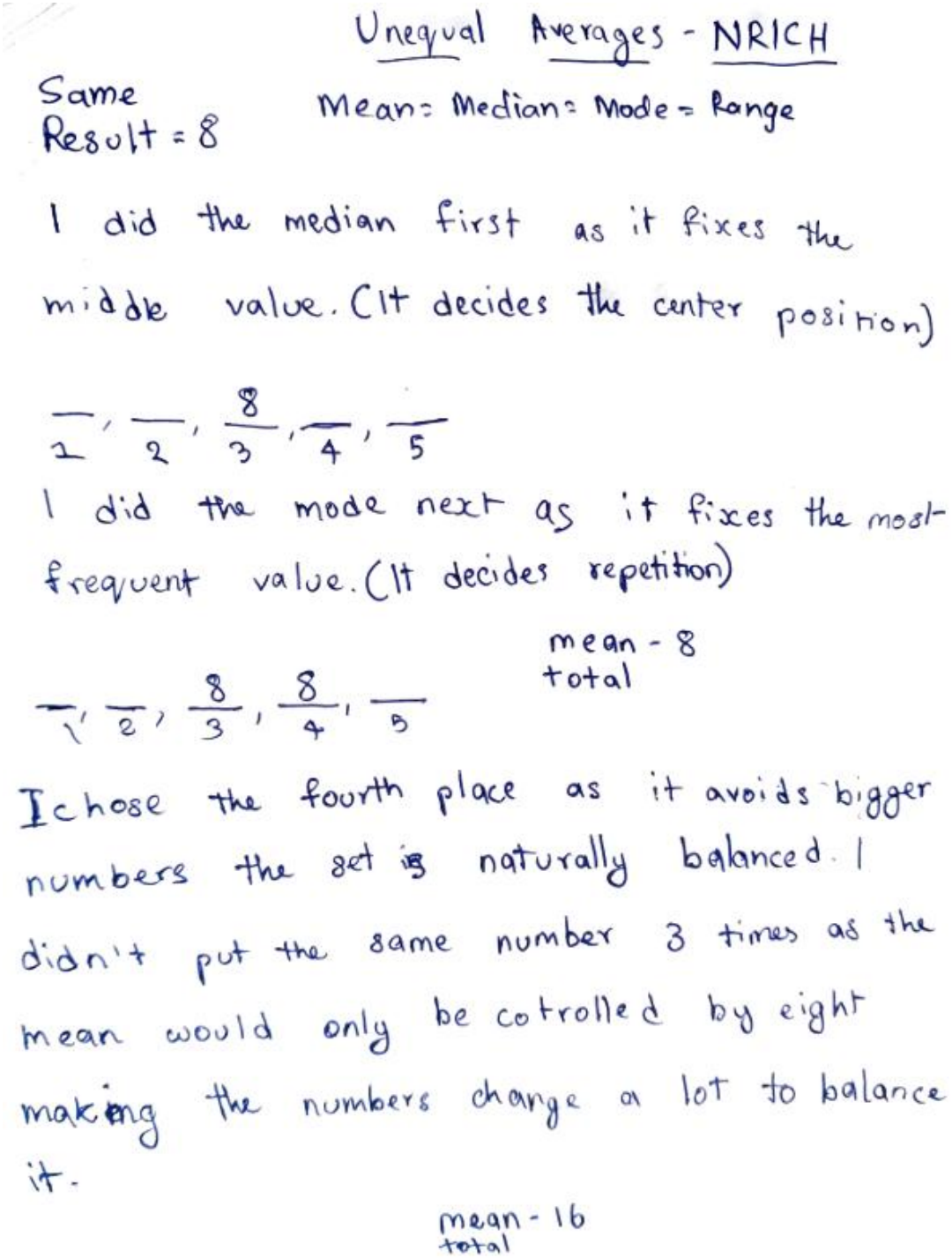
Solution for Unequal averages NRICH secondary live problem

Hello NRICH team,

This is Meera Sriram from Chennai, India. I worked on Unequal averages problem (https://nrich.maths.org/problems/unequal-averages) NRICH secondary live problem as part of the STEPS IN MATH program conducted by The GYM Foundation. (https://thegymfoundation.com/stepsinmath)

Here is the scanned copy of my solution.



Unequal Averages - NRICH

Same Result = 8

Mean = Median = Mode = Range

I did the median first as it fixes the middle value. (It decides the center position)

_, _, 8, _, _

I did the mode next as it fixes the most-frequent value. (It decides repetition)

mean - 8
total

_, _, 8, 8, _

I chose the fourth place as it avoids bigger numbers the set is naturally balanced. I didn't put the same number 3 times as the mean would only be controlled by eight making the numbers change a lot to balance it.

mean - 16
total

We also have to check the mean total after every step as the sum changes so the mean can also change. The mean total should be 40 so that $\frac{40}{5} = 8$.

For the range, I made a table based on the possibilities of numbers there can be -

0	8
2	10
3	11
4	12
5	13

It cannot have the pair 14, 6 or more because their sum with the other numbers is 36, leaving 4 in the 2nd place number. But 4 is smaller than 6, which was supposed to be the smallest number for range. (in this case)

$\frac{5}{1}, \frac{\quad}{2}, \frac{8}{3}, \frac{8}{4}, \frac{13}{5}$

mean total - ~~38~~ 34

40 - 34 = 6 so the 2nd place number should be 6.

5, 6, 8, 8, 13

Check

Median = 8 (middle number)

Mode = 8 (appears twice)

Mean = $\frac{5+6+8+8+13}{5} = \frac{40}{5} = 8$

Range = 13 - 5 = 8

5, 6, 8, 8, 13 is just one set but there can be multiple sets that satisfy the rule of having the same number as mean, mode, median, range. Another example is 3, 8, 8, 10, 11

A. Mode < Median < Mean

I started with mode as mode should be the smallest number in the set so mode = 1

$\frac{1}{1}, \frac{1}{2}, \frac{\ }{3}, \frac{\ }{4}, \frac{\ }{5}$

Next is the median. Median should be greater than 1 so median = 3.

$\frac{1}{1}, \frac{1}{2}, \frac{3}{3}, \frac{\ }{4}, \frac{\ }{5}$

The total for mean should be a multiple of 5. When the total is divided by 5, it should be greater than 3 so I chose 20 as the total.

1+1+3 = 5 20-5 = 15. The other 2 numbers should add up to 15. 6+9 = 15

1, 1, 3, 6, 9

Check

$1+1+3+6+9 = \frac{20}{5} = 4 > 3 > 1$

B. Mode < Mean < Median

Mode should be the smallest so mode = 1.

$\frac{1}{1}, \frac{1}{2}, \frac{\ }{3}, \frac{\ }{4}, \frac{\ }{5}$

Median should be gretest, so I chose higher values.

$\frac{1}{1}, \frac{1}{2}, \frac{10}{3}, \frac{\ }{4}, \frac{\ }{5}$

The mean should be smaller than 10 so mean = 7.

If mean = 7, mean total = $7 \times 5 = 35$. $10 + 1 + 1 = 12$

$35 - 12 = 23$ 23 should be the sum of the other 2 numbers. The other 2 numbers should be greater than $10. 11 + 12 = 23$ So —

1, 1, 10, 11, 12

Check

$10 > \frac{1+1+10+11+12}{5} = \frac{35}{5} = 7 > 1$

C. Mean < Mode < Median

$\frac{\quad}{1}, \frac{\quad}{2}, \frac{n}{3}, \frac{\quad}{4}, \frac{\quad}{5}$

median = n

As Mode < Median, the maxium mode can be is n-1.

$\frac{n-1}{1}, \frac{n-1}{2}, \frac{n}{3}, \frac{\quad}{4}, \frac{\quad}{5}$

The minimum the other 2 numbers(4,5) can be is n+1 and n+2.

$\frac{n-1}{1}, \frac{n-1}{2}, \frac{n}{3}, \frac{n+1}{4}, \frac{n+2}{5}$

$$\text{mean} = \frac{(n-1)+(n-1)+n+(n+1)+(n+2)}{5} =$$

$$\frac{5n+1}{5} = \frac{5n}{5} + \frac{1}{5} = n+\frac{1}{5}$$

The mean should be smaller than mode so mean should be smaller than n-1 but it cannot be as it atleast should be $n+\frac{1}{5}$.

So it is not possible to have Mean < Mode < Median. in a set having five numbers.