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My Extended Solution

Things to notice straight away

- The triangle created must be isosceles.
  ∴ two sides must be equal, so the apex must have the midpoint of the x or y value (one of them) of the base line segment e.g.

- The area must be 9 cm².
  ∴ The area of the triangle is $b \times h \times \frac{1}{2}$. So, if the area is 9, $b \times h \times \frac{1}{2} = 9$ which means, if you muegltiply both sides by 2, $b \times h = 18$. That means the base and height must be factor pairs of 18.
- One of the vertices must be at point (20, 20)
  ∴ That means (20, 20) must be part of the base or apex.

Working Out

I split my working out into two sections: if (20,20) is part of the base (Part A) or if (20, 20) is the apex (Part B).

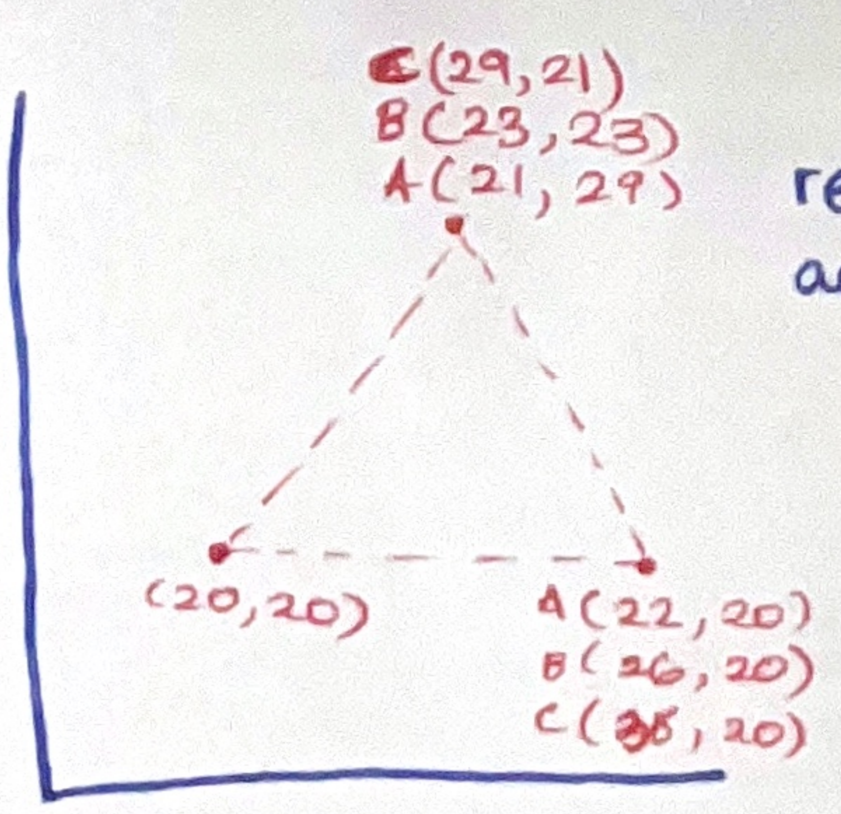
Part A

Because the apex is above the base or across, the base must have an even length, to keep integer coordinates and to reinforce the isosceles structure of the triangle.

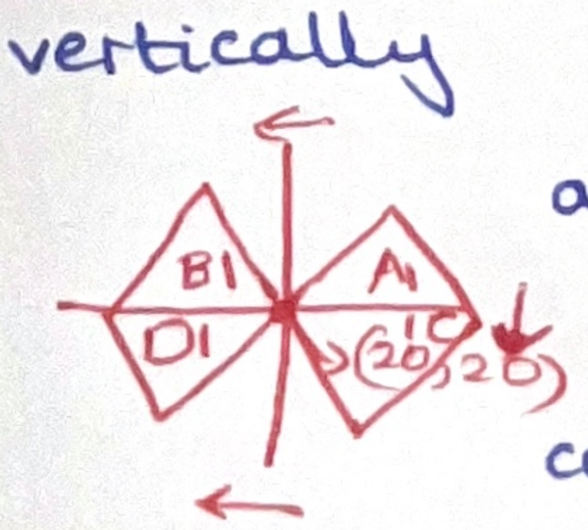
Factor pairs of 18 but base must be even.

base	height	
1	18	× odd base
2	9	
3	6	× odd base
6	3	
9	2	× odd base
18	1	

We should also account for both vertical and horizontal bases.

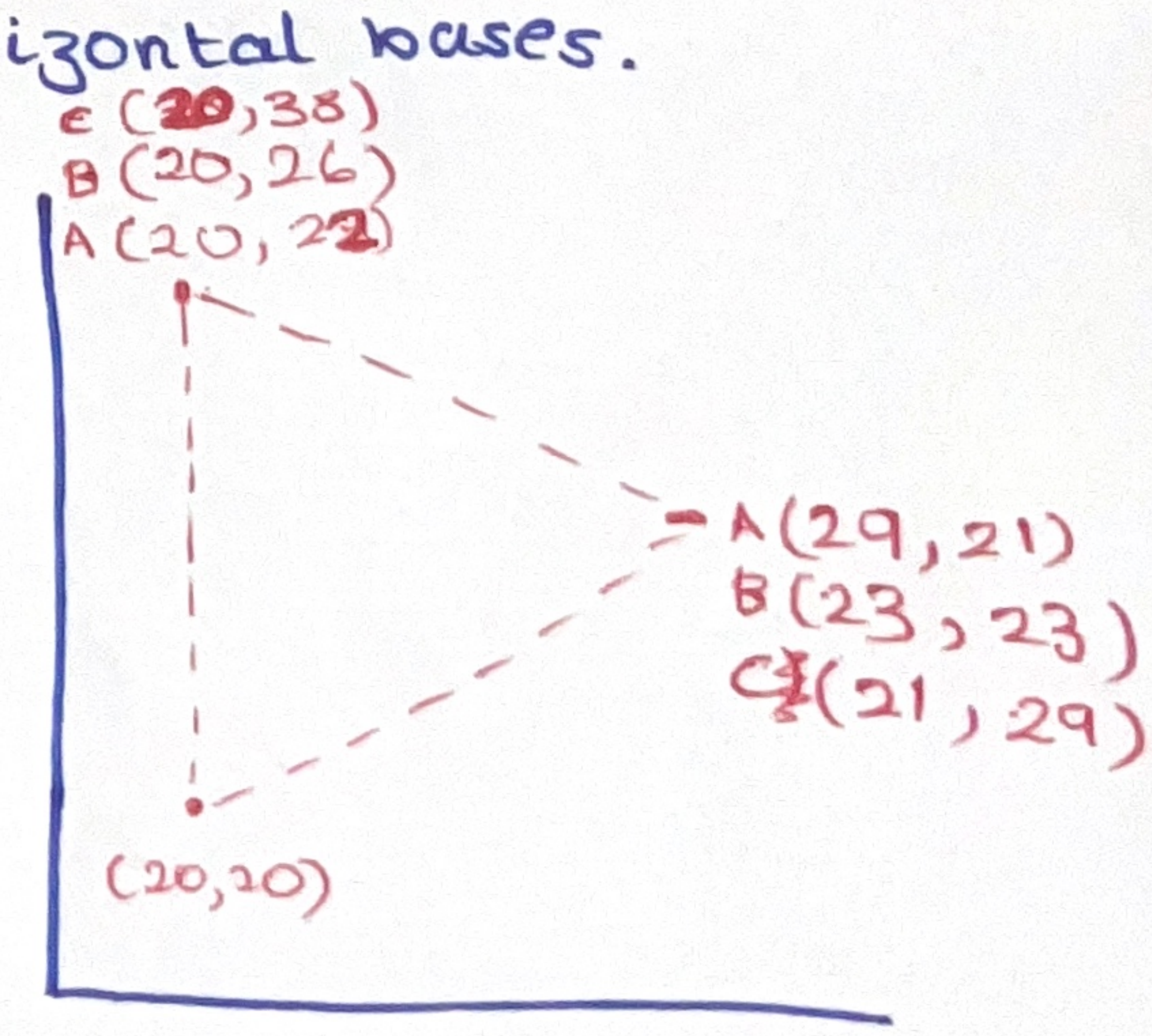


but we can reflect it horizontally and vertically

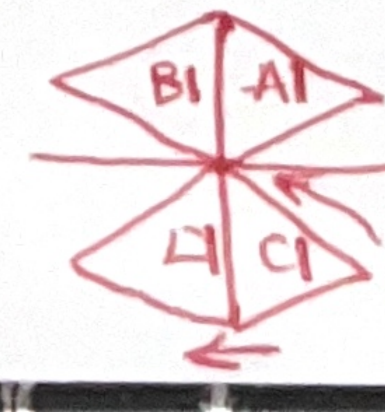


so horizontal possibilities are 3 × 4 = 12 🙂

but don't forget the congruent vertical base triangles...



so vertical possibilities are 3×4=12.

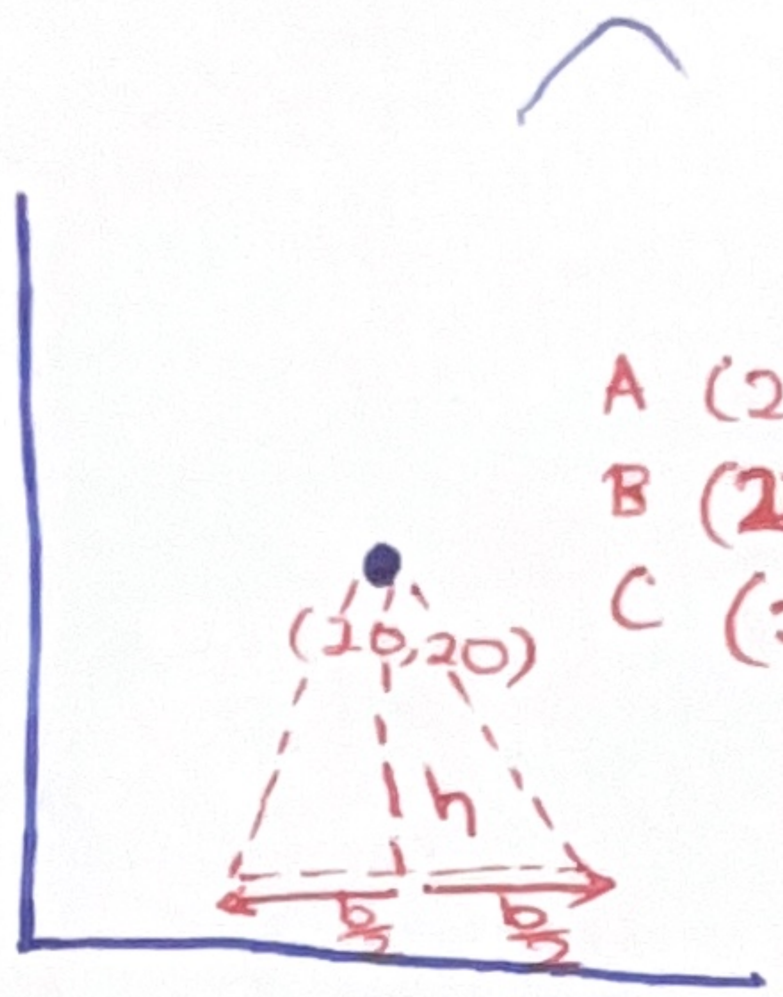


In conclusion, total for Part A is 12 + 12 = 24 triangles

Part B

Here, (20,20) is the ~~basic~~ apex of the isosceles triangle. Then (20,20) must be the midpoint of either the x or y value. And again the base must be even to have an integer midpoint value.

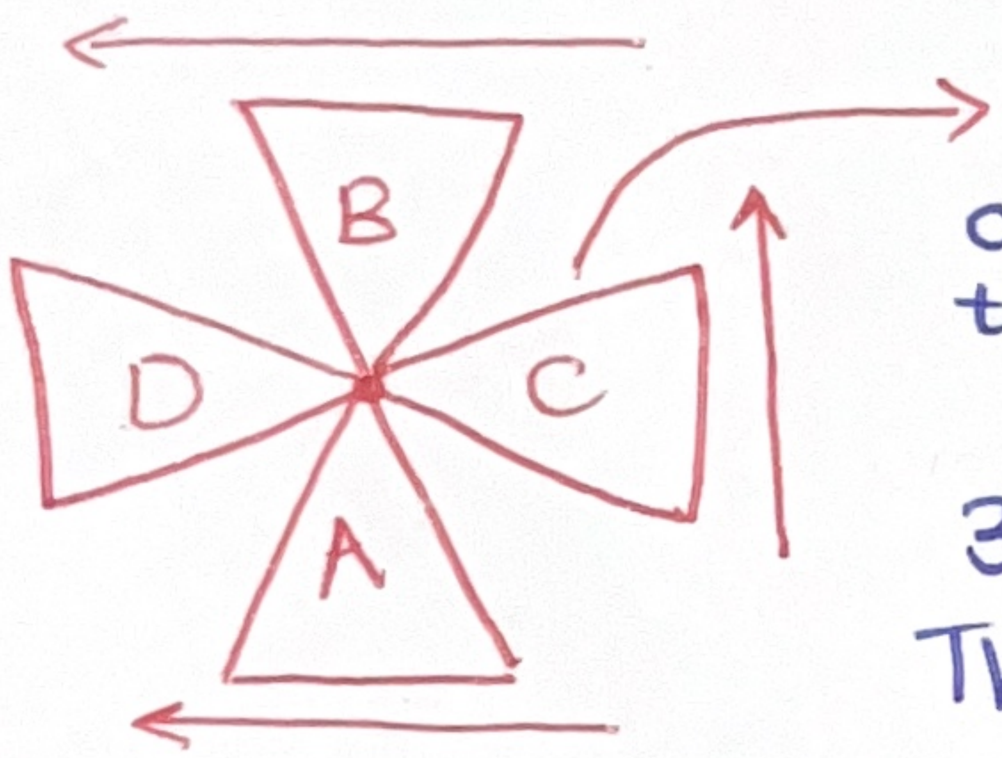
Base	Height
2	9
6	3
18	1



A (20,20) A(19,11) A(21,11)
B (20,20) B(17,17) B(23,17)
C (20,20) C(11,19) C(29,19)

Notice:
→ The y value of the base is the same here; they're on the same horizontal line
→ The coordinates of the base is the same distance from the apex. Isosceles has 1 line of symmetry from the mid-point of the base cutting through the apex.

We should also take into account reflections.



You could also do it from a different side still perpendicular to (20,20) of the midpoint

$3 \times 4 = 12$

That's the total for Part B.

In conclusion, total for Part B is 12 triangles.

Conclusion: Part A + Part B
= 24 + 12
= 36 triangles in total.

∴ 36 possibilities fitting the criteria.

Thank you!! By Lamees Ede, Doha College, age 13

P.S. I counted every triangle with different coordinates as a seperate triangle.