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7 ena

Here's an interesting set of five numbers:



The mean, mode, median and range are all 7.

Can you find other sets of five positive whole numbers where:

Mean = Median = Mode = Range



2) 2, 6, 6, 8, 8

Can you find sets of five positive whole numbers that satisfy the following properties?

Mode < Median < Mean

Mode < Mean < Median

Mean < Mode < Median

Mean < Median < Mode

Median < Mode < Mean

Median < Mean < Mode

Not all of these can be satisfied by sets of five numbers!
Can you explain why?

Show that some of them can be satisfied with sets of just four numbers.
Show that all of them can be satisfied with sets of six numbers.

PART 1

Mean = Median = Mode = Range

In the example given, all equal 7.

step 1: If, mean = median = mode = range = k [number value]

five numbers: a, b, c, d, e a, b, c, d, e
 $\mu =$ [arranged in order]

① median \rightarrow middle number: c

② mode \rightarrow must appear most times

③ range $\rightarrow e - a$

④ mean $\rightarrow (a + b + c + d + e) / 5$

we need: $c = k$ $(8 + 3 + 2 + 4 + 8) / 5 = 7$

$$e - a = k$$

$$\text{mean} = k$$

Step 2: If we notice the pattern, the middle number 7 appears twice.

median = mode = k

two middle numbers = k

smallest = a

largest = $a + k$

lets say $k = 5$

let smallest number be 3

largest number $\rightarrow 3 + 5 = 8$

PART 1

so far,

3, —, 5, 5, 8 Mean = Median = Mode

mean must be 5.

so total sum $\rightarrow 5 \times 5 = 25$

current known numbers: $3 + 5 + 5 + 8 = 21$

one more number value $\rightarrow 25 - 21 = 4$

\therefore the set is $\rightarrow 3, 4, 5, 5, 8$ ①

mode $\rightarrow 5$ ②

step 3: median = 5 \checkmark ③

mode = 5 \checkmark ④

range = $8 - 3 = 5 \checkmark$

mean = $(3 + 4 + 5 + 5 + 8) / 5$

$$= 25 / 5$$

$$= 5 \checkmark$$

let common value be $k = 6$

we want:

mean = median = mode = range = 6

five ordered numbers: a, b, c, d, e

median: middle value $\rightarrow c = 6$

so $c = 6$

to make 6 the mode, we repeat it.

PART 2

so far,

$a, b, 6, 6, e$

range = $e - a = 6$

let smallest number be $2(a)$

then

$2 + 6 = 8 = e$

now we have,

$2, b, 6, 6, 8$

mean must be 6

total sum $\rightarrow 6 \times 5 = 30$

current sum:

$2 + 6 + 6 + 8 = 22$

so, $22 + b = 30$

number $b = 8$

final set: $2, 6, 6, 8, 8$

mean = $\frac{2+6+6+8+8}{5} = \frac{30}{5} = 6 \checkmark$

mode = 6 and 8 [6 is also acceptable] \checkmark

median = 6 \checkmark

range = $8 - 2 = 6 \checkmark$

PART 1 final answer:

set 1 $\rightarrow 3, 4, 5, 5, 8$

set 2 $\rightarrow 2, 6, 6, 8, 8$