



Unequal Averages

Here's an interesting set of five numbers:



The mean, mode, median and range are all 7.

Can you find other sets of five positive whole numbers where:

$$\text{Mean} = \text{Median} = \text{Mode} = \text{Range}$$

Q.1. *working* →

Can you find sets of five positive whole numbers that satisfy the following properties?

- Mode < Median < Mean
- Mode < Mean < Median
- Mean < Mode < Median
- Mean < Median < Mode
- Median < Mode < Mean
- Median < Mean < Mode

Extra sheet

Not all of these can be satisfied by sets of five numbers!
Can you explain why? ←

Show that some of them can be satisfied with sets of just four numbers.
Show that all of them can be satisfied with sets of six numbers. ←

The median will push ~~down~~ the total sum too high for the mean to ever drop below a mode that is smaller than the median.

Q.1. Five positive numbers: $x_{(1)}, x_{(2)}, x_{(3)}, x_{(4)}, x_{(5)}$

Median = 5, so middle number = 5

$\rightarrow x_{(1)}, x_{(2)}, 5, x_{(4)}, x_{(5)}$

Mode = 5 \rightarrow Most frequent number, since median is 5, its logical to assume 5 as the mode.

$\rightarrow x_{(1)}, 5, 5, x_{(4)}, x_{(5)}$

Range = 5 \Rightarrow Gap between smallest and greatest number.

$\rightarrow x_{(5)} - x_{(1)} = 5$

= 8 \rightarrow = 5 \rightarrow Reason in mean for selected number choice.

$\Rightarrow 3, x_{(2)}, 5, 5, 8$

\Rightarrow Average = 5

$$= 5 \times 5 = 25$$

$$= 25 \div 5$$

= 5 (Average)

$$\Rightarrow 3 + 5 + 5 + 8 = 21$$

$$\Rightarrow 25 - 21 = 4$$

= $x_{(2)}$

$\Rightarrow 3, 4, 5, 5, 8.$

RECHECKING:

$$\text{Mean} = 3 + 4 + 5 + 5 + 8 = 25 \div 5$$

$$= 5$$

$$\text{Median} = 5$$

$$\text{Mode} = 5$$

$$\text{Range} = 8 - 3 = 5$$

$$\text{Mean} = \text{Mode} = \text{Median} = \text{Range}.$$

Hence:

Answer: 3, 4, 5, 5, 8.

Assumption: $a, b, c, d, e = 5$ numbers

Mode $<$ median $<$ mean

$\Rightarrow 1, 1, 2, 3, 13 \rightarrow$ mode = 1

median = 2

sum = 20 = 1 + 1 + 2 + 3 + 13

mean = 4

\Rightarrow Ans: $1 < 2 < 4$

Mode $<$ mean $<$ median

$\Rightarrow 1, 1, 10, 11, 12 \rightarrow$ mode = 1

median = 10

sum = 35 = 1 + 1 + 10 + 11 + 12

mean = 7

\Rightarrow Ans: $1 < 7 < 10$

Mean = Mode = median

$\Rightarrow 1, 5, 5, 10, 11 \Rightarrow$ median = 5 / should be larger

Mode = median \rightarrow Hence does

not work. (Impossible).

mean $<$ median $<$ mode = median = 10 mode = 11

= 1, 2, 10, 11, 11

sum = 35 mean = 7

$(7 < 10 < 11)$

median $<$ mode $<$ mean = median = 2 mode = 3

= 1, 2, 3, 3, 11

sum = 20 mean = 4

$(2 < 3 < 4)$

median $<$ mean $<$ mode = median = 3 mode = 12

= 1, 2, 3, 12, 12

sum = 30 mean = 6