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Unequal Averages

Here's an interesting set of five numbers:

The mean, mode, median and range are all **7**.

Can you find other sets of five positive whole numbers where:

Mean = Median = Mode = Range

1) 3 4 5 5 8

2) 2 4 4 4 6

Can you find sets of five positive whole numbers that satisfy the following properties?

Mode < Median < Mean

Mode < Mean < Median

Mean < Mode < Median

Mean < Median < Mode

Median < Mode < Mean

Median < Mean < Mode

Not all of these can be satisfied by sets of five numbers!
Can you explain why?

Show that some of them can be satisfied with sets of just four numbers.
Show that all of them can be satisfied with sets of six numbers.

Set 1

common value be k

If mean = median = mode = range = k

let the 5 numbers be a, b, c, d, e

median = k

mode = k

mean = (a+b+c+d+e)/5 = k

range = e − a

= k

c = k

smallest = a

largest = a+k

lets take k = 5

Let smallest number = 3

largest number = 5+3

= 8

so far

3, ?, 5, 5, 8

Now lets go to mean

mean must be 5

5 × 5 = 25

current known numbers

3+5+5+8 = 21

25 − 21 = 4

So final set

= 3, 4, 5, 5, 8

mean = $\frac{3+4+5+5+8}{5}$

= $\frac{25}{5}$

= 5

range = 8 − 3 = 5

mode = 5

median = 5

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Set 2

common value be k

If mean = mode = range = median = k

let 5 numbers be a, b, c, d, e

median = k

mode = k

mean = (a+b+c+d+e)/5 = k

range = e − a

= k

c = k

smallest = a

largest = a + k

lets take k = 4

let smallest number ~~or be~~ 2

largest number = 2 + 4 = 6

so far

2, ?, 4, 4, 6

Now lets go to mean

mean must be 4

4 × 5 = 20

current known numbers

2 + 4 + 4 + 6 = 16

20 − 16 = 4

Final Set = 2, 4, 4, 4, 6

range = 6 − 2 = 4

mode = 4

median = 4

mean = (2 + 4 + 4 + 4 + 6)/5

= 20/5 = 4

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① Mode < Median < Mean

example - 1, 1, 3, 5, 8

mode = 1 Median 3 Mean = 18/5 = 3.6

1 < 3 < 3.6

② Mode < Mean < median

Example - 1, 1, 6, 7, 8

. mode = 1 Median 6 Mean = 23/5 = 4.6

1 < 4.6 < 6

③ Median < mode < Mean

Ex - 1, 2, 4, 4, 9

Median = 4 mode = 4 Median = mode

so adjust 1, 2, 3, 4, 4, 9. — 6 numbers

so ordering is hard 5 numbers

Why 5 numbers not possible.

Because median is the middle value

Mean < Median < Mode

For mode to be largest, the largest value must repeat.

Some work with 4 numbers.

Mean < Median < Mode

1, 4, 5, 5 Mode = 5

Median = 4.5

Mean = 3.75

All work with 6 numbers

Mean < Median < Mode

1, 2, 4, 5, 9, 9 Mode = 9 Median 4.5

Mean 5.