① To find sets of 10 consecutive numbers where

a) $\frac{n-9}{1} \in \mathbb{Z}$
b) $\frac{n-8}{2} \in \mathbb{Z}$
c) $\frac{n-7}{3} \in \mathbb{Z}$
d) $\frac{n-6}{4} \in \mathbb{Z}$
e) $\frac{n-5}{5} \in \mathbb{Z}$
f) $\frac{n-4}{6} \in \mathbb{Z}$
g) $\frac{n-3}{7} \in \mathbb{Z}$
h) $\frac{n-2}{8} \in \mathbb{Z}$
i) $\frac{n-1}{9} \in \mathbb{Z}$
j) $\frac{n}{10} \in \mathbb{Z}$

Now that we have a formula, we can plug in $n_1 = 10$,

$$n_1 = 10 \left(\frac{1}{1}=1,\ \frac{2}{2}=1,\ \frac{3}{3}=1,\ \frac{4}{4}=1,\ \frac{5}{5}=1,\ \frac{6}{6}=1,\ \frac{7}{7}=1,\ \frac{8}{8}=1,\ \frac{9}{9}=1,\ \left(\frac{10}{10}=1\right) \right)$$

$$n_2 = LCM(10,9,8\ldots1) \times 2 - (LCM(10,9,8\ldots1) - 10)$$
$$= 5040 - (2510) = 2530 \left(\frac{2521}{1} = 2521,\ \frac{2522}{2} = 1261,\ \frac{2523}{3} = 841,\ \frac{2524}{4} = 631,\ \frac{2525}{5} = 505,\ \frac{2526}{6} = 421,\ \frac{2527}{7} = 361,\ \frac{2528}{8} = 316,\ \frac{2529}{9} = 281 \right)$$

$$n_3 = LCM(10 \to 1) \times 3 - (LCM(10 \to 1) - 10)$$
$$= 7560 - 2510 = 5050$$ (Also works for values $1 \to 10$ but I don't have enough room)

You can find more iterations of n, but this doesn't give EVERY solution.

Firstly, we should start with a simpler problem where

a) $\frac{n-2}{2} \in \mathbb{Z}$
b) $\frac{n-1}{3} \in \mathbb{Z}$
c) $\frac{n}{4} \in \mathbb{Z}$

We can see that the first n should be 4 here:

$$\left(\frac{4-2}{2} = 1 ✓\right), \left(\frac{4-1}{3} = 1\right)✓, \left(\frac{4}{4} = 1\right)✓$$

$\therefore n_1 = 4$

If we brute force the next 2 answers we get

$$n_2 = 16 \Rightarrow \left(\frac{16-2}{2} = 7\right)✓, \left(\frac{16-1}{3} = 5\right)✓, \left(\frac{16}{4} = 4\right)✓$$

$$n_3 = 28 \Rightarrow \left(\frac{28-2}{2} = 13\right)✓, \left(\frac{28-1}{3} = 9\right)✓, \left(\frac{28}{4} = 7\right)✓$$

With our first 3 answers of 4, 16 and 28, we can make a formula. Since they all have a difference of 12, the formula would be $\boxed{n_x = 12x - 8}$ where x is the iteration of n. We can see here that x is multiplied by 12. 12 is also the LCM of 2, 3 and 4, and -8 is 12 − the first iteration of n_1 which is usually the highest multiple requirement.

$\therefore$ The new formula would be

$$n_x = LCM(n_1, n_1 - 1, \ldots 1)(x) - (LCM(n_1, n_1 - 1 \ldots 1) - n_1)$$