

Negative Powers

Question:

$$\left((-4)^{-3}\right)^{-2}$$

Step 1: $(-4)^{-3}$ means $-(4^{-3})$
 $= (-1) \times \frac{1}{4^3}$

Step 2: $\left((-1) \times \frac{1}{4^3}\right)^2$ means $\frac{1}{\left((-1) \times \frac{1}{4^3}\right)^2}$

$$= \frac{1}{(-1)^2 \times \left(\frac{1}{4^3}\right)^2}$$

$$= \frac{1}{(-1)^2 \times \frac{1}{(4^{3 \times 2})}}$$

$$= \frac{1}{(-1)^2 \times \frac{1}{4^{3 \times 2}}}$$

$$= \frac{4^{3 \times 2}}{(-1)^2}$$

Step 3: $\left(\frac{4^{3 \times 2}}{(-1)^2}\right)^{-1}$ means $\frac{1}{\left(\frac{4^{3 \times 2}}{(-1)^2}\right)^1}$

$$\left(\frac{4^{3 \times 2}}{(-1)^2}\right)^1$$

$$= \frac{(-1)^{2 \times 1}}{(4^{3 \times 2})^1}$$

$$\therefore \left((-4)^{-3}\right)^{-2} = \frac{(-1)^{2 \times 1}}{(4^{3 \times 2})^1}$$

Order of powers	Expression	Interpretation	Result
-4 -3 -2 -1	$((-4^{-3})^{-2})^{-1}$	$\frac{(-1)^{2 \times 1}}{(4^{3 \times 2})^1}$	$\frac{+1}{4^{3 \times 2 \times 1}}$
-4 -2 -3 -1	$((-4^{-2})^{-3})^{-1}$	$\frac{(-1)^{3 \times 1}}{(4^{2 \times 3})^1}$	$\frac{-1}{4^{2 \times 3 \times 1}}$
-4 -3 -1 -2	$((-4^{-3})^{-1})^{-2}$	$\frac{(-1)^{1 \times 2}}{(4^{3 \times 1})^2}$	$\frac{+1}{4^{2 \times 3 \times 1}}$
-4 -2 -1 -3	$((-4^{-2})^{-1})^{-3}$	$\frac{(-1)^{1 \times 3}}{(4^{2 \times 1})^3}$	$\frac{-1}{4^{2 \times 1 \times 3}}$
-4 -1 -3 -2	$((-4^{-1})^{-3})^{-2}$	$\frac{(-1)^{3 \times 2}}{(4^{1 \times 3})^2}$	$\frac{+1}{4^{1 \times 3 \times 2}}$
-4 -1 -2 -3	$((-4^{-1})^{-2})^{-3}$	$\frac{(-1)^{2 \times 3}}{(4^{1 \times 2})^3}$	$\frac{+1}{4^{1 \times 2 \times 3}}$

$(-1)^{\text{EVEN}} \rightarrow \text{EVEN}$
 $(-1)^{\text{ODD}} \rightarrow \text{ODD}$
 THEN $(-1)^{\text{EVEN}} = +1$
 THEN $(-1)^{\text{ODD}} = -1$

$((-4^{-2})^{-3})^{-1} = \frac{(-1)^{3 \times 1}}{4^{2 \times 3 \times 1}}$
 $((-4^{-3})^{-1})^{-2} = \frac{(-1)^{1 \times 2}}{4^{3 \times 1 \times 2}}$
 $((-4^{-1})^{-3})^{-2} = \frac{(-1)^{3 \times 2}}{4^{1 \times 3 \times 2}}$
 $((-4^{-1})^{-2})^{-3} = \frac{(-1)^{2 \times 3}}{4^{1 \times 2 \times 3}}$
 IF $\star = 1 \Rightarrow (-1)^{\star} = -1$

In this case, there are only two values: $\frac{+1}{4096}$ and $\frac{-1}{4096}$

Means two possible values.

When

x is even then

$$\frac{(-1)^x}{1} = \frac{1}{1} \text{ equals } +1$$

$$\frac{(-1)^x}{1} = \frac{-1}{1} \text{ equals } -1$$

When

x is odd then

$$(-1)^x = -1$$

This expression has a range value between 1 and -1.

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