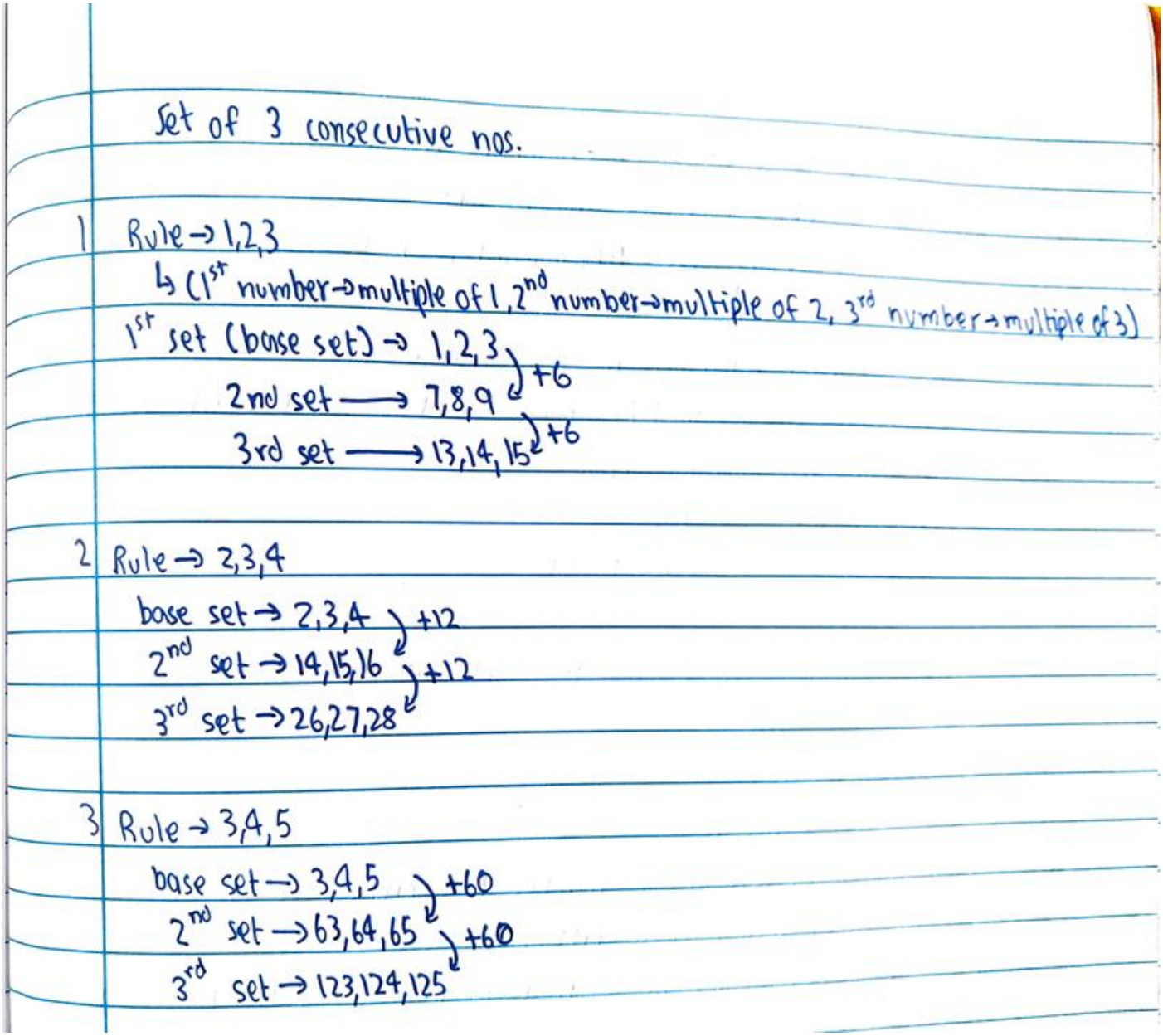
Solution for Secondary live problem Multiple Surprises from NRICH

Hello NRICH team,

This is Barathraam Gurumoorthy Sriram from Chennai, India.

I worked on Multiple surprises (https://nrich.maths.org/problems/multiple-surprises) NRICH secondary live problem as part of the STEPS IN MATH program conducted by The GYM Foundation. (https://thegymfoundation.com/stepsinmath)

Please find my scanned solution below for the Multiple surprises problem.



Set of 4 consecutive nos.

1 Rule → 1,2,3,4

base set → 1,2,3,4 ⟩ +12

2nd set → 13,14,15,16 ⟩ +12

3rd set → 25,26,27,28

2 Rule → 2,3,4,5

base set → 2,3,4,5 ⟩ +60

2nd set → 62,63,64,65 ⟩ +60

3rd set → 122,123,124,125

3 Rule → 3,4,5,6

base set → 3,4,5,6 ⟩ +60

2nd set → 63,64,65,66 ⟩ +60

3rd set → 123,124,125,126

Thought - 'How are the ~~not~~ numbers in 2 consecutive sets related?'

(Eg → base set and 2nd set)

→ difference = LCM of numbers in base set

↓

between 2 numbers in 2 consecutive sets

Conjecture - adding the LCM of number in base set to a set gives the next set

Applying in original problem...

base set = 1,2,3...8,9,10

= LCM of all numbers 1 to 10

= 2520

→ numbers in 2 consecutive sets differ by 2520

Another intuition-

Rule → 1 to 10

'We need to add a number to current set (to get another set) so that rule still holds.'

'Only way we can do that is by adding the LCM of base-set, since then → every number in set_n + LCM = divisible by the number.'

So, with base-set of numbers 1 to 10, LCM = 2520 → we have to add 2520 to each set to get the next set.

So, all sets can be found using-

$Set_n = 2520n + k$

↳ the number in base-set (1 to 10)

example-

$2520 \cdot 0 + 1 \rightarrow 1$
...
$2520 \cdot 0 + 10 \rightarrow 10$
} first set: 1 → 10

$2520 \cdot 1 + 1 \rightarrow 2521$
...
$2520 \cdot 1 + 10 \rightarrow 2560$
} second set = 2521 → 2560

Generalisation -

'm' number of consecutive nos.

set $m = \{m_1, m_2, m_3 \dots m_n\}$ ← base set

L = LCM of numbers in set m → $LCM(m_1, m_2, m_3 \dots m_n)$

: 1st set = $m_1, m_2, m_3 \dots m_n$

2nd set = $m_1 L, m_2 L, m_3 L \dots m_n L$

3rd set = $m_1 \cdot 2L, m_2 \cdot 2L, m_3 \cdot 2L \dots m_n \cdot 2L$

: base set [k] + nL ← set number (1st set, 2nd set, 3rd set...)

number in set m ($m_1/m_2/m_3 \dots$) ↵ → LCM of numbers in base set

in this case -

LCM of 1→10

Simplifying ~~prb~~ problem by identifying repeatitions →

10 → 5x2 → 2 and 5 eliminated

9 → 3x3 → 3 eliminated

8 → 4x2 → 4 eliminated

7 → nothing

6 → 3x2

5 → nothing → x

4 → 2x2 → x

3 → nothing → x

2 → nothing → x

~~ma~~ factors of 6 → 3 and 2 (which are eliminated)

: 6 is eliminated

: LCM (7,8,9,10) → LCM (8,9,10)·7 → LCM (72,10)·7 → 360·7 = 2520 ← 7 is prime : no impact

: $2520_n + k$