

Solution for Less is More NRICH primary live problem

Hello NRICH team,

This is Meera Sriram from Chennai, India.

I worked on Less is More (<https://nrich.maths.org/problems/less-more>) NRICH primary live problem as part of the STEPS IN MATH program conducted by The GYM Foundation.

(<https://thegymfoundation.com/stepsinmath>)

Please find my scanned copy of handwritten solution below for the Less is More problem. It was a very interesting game to find the highest score and to find my own strategy to form numbers that get highest score. I also understood why my strategy will not work for some set of numbers.

Nrich - Less is more

At first, I did trial and error- but with much thinking I developed a strategy.

First, I came up with a normal sequence $\rightarrow 1, 2, 3, 4, 5, 6, 7, 8$ and arranged them using trial and error.

$$\begin{array}{r} 65 < 78 \\ 21 < 43 \\ \hline 86 \end{array}$$

$$\begin{array}{r} 45 < 67 \\ 81 < 82 \\ \hline 76 \end{array}$$

$$\begin{array}{r} 73 < 81 \\ 54 < 62 \\ \hline 127 \end{array}$$

And found
the highest

Then I tried to observe the number carefully and figure out the logic.

My logic

h = highest s = smallest

I divided the 8 numbers into 2 groups

s smallest group

1, 2, 3, 4
↓ ↓ ↓ ↓
s₄ s₃ s₂ s₁

h highest group

5, 6, 7, 8
↓ ↓ ↓ ↓
h₄ h₃ h₂ h₁

And write the position of each number in the game from each group

Approach to form 2-digit numbers

$$73 < 81$$

$$54 < 62$$

$$h2, s2 < h1, s4$$

$$h4, s1 < h3, s3$$

This approach wouldn't work if it was in random order, like 3, 8, 2, 1, 5, 7, 6, 4. So I realised, the numbers should be arranged in ascending order at the first itself.

Steps to find four 2-digit numbers

$$N1 < N2$$

$$N3 < N4$$

1) Arrange the numbers in ascending order.

2) Divide the first four numbers as 's' group, and the other four numbers as 'h' group.

's'

$$s4, s3, s2, s1$$

'h'

$$h4, h3, h2, h1$$

3) The highest four numbers should be placed in the tens place value, the smallest four numbers should be placed in the ones place value like below.

check

$$1) 5, 3, 9, 7, 0, 2, 1, 4$$

$$0, 1, 2, 3, 4, 5, 7, 9$$

$$72 < 90$$

$$43 < 51$$

$$\boxed{115} = \text{highest}$$

It works but what if the numbers repeat?

$$2) 5, 5, 6, 5, 4, 4, 1, 6$$

$$1, 4, 4, 5, 5, 5, 6, 6$$

$$64 < 61$$

$$55 < 54$$

Oh, it does not work because the tens are mostly $\boxed{115}$ = highest same and the ones place value and its digits matter the most. So in this case, the approach should be -

$$h2, s4 < h1, s2$$

$$h4, s3 < h3, s1$$

$$h2, s2 < h1, s4$$

$$h4, s1 < h3, s3$$

check

$$5, 5, 6, 5, 4, 4, 1, 6$$

$$1, 4, 4, 5, 5, 5, 6, 6$$

$$61 < 64$$

$$54 < 55$$

$$\boxed{115} = \text{highest}$$

When all the number except one is same, the rule cannot be satisfied.