

Hi Nrich Team,

I am Preethi Rao. I am one of the facilitators at '**Ganit Kreeda**', **Vichar Vatika, India**. I worked collaboratively with a group of students from 4th grade to 7th grade, online, on "**KEEP IT SIMPLE**". The names of the students are:

Anirved Mahanta, Aarya Kathe, Abheer Deo, Ashaman Singh, Dia Ramalingam, Mihika Kothari, Reyansh Sethumadhavan, Dhanwin, Darsh Rathi, Shiva N Pant, Siddhanth Adahavan, Sunil Patil

Children observed that:

1. **Can you describe Charlie's rule?** - One of the addends' denominators must be a product of the other addend's denominator and the sum's denominator.

2. **Are all his examples correct?** - No, the second one and the fourth one are not correct.

3. **What do you notice about the sums that are correct?** - The addends' denominator and the given sum's denominator are consecutive numbers.

4. **Find some other correct examples..**

$$\frac{1}{5} = \frac{1}{6} + \frac{1}{30}$$

$$\frac{1}{6} = \frac{1}{7} + \frac{1}{42}$$

5. **How would you explain to Charlie how to generate lots of correct examples?** - Take two consecutive numbers and multiply them together. Ex.  $7 \times 8 = 56$ .

Write the consecutive numbers and the product as unit fractions.

Write the sum of the two smaller fractions as the sum of the biggest fraction.

I.e  $1/7 = 1/8 + 1/56$

Most of the children generalised the above as:

The image shows three handwritten steps for generalizing the fraction decomposition:

- Step 1:**  $\frac{1}{a}$  write a unit fraction
- Step 2:**  $\frac{1}{a} = \frac{1}{a+1} + x$  with the successor.
- Step 3:**  $\frac{1}{a} = \frac{1}{a+1} + \frac{1}{a(a+1)}$

6. Charlie tried to do the same with  $\frac{1}{8}$ . Can you finish Charlie's calculations to see which ones work?

$$\frac{1}{8} = \frac{1}{9} + \frac{1}{72}$$

$$\frac{1}{8} = \frac{1}{10} + \frac{1}{40}$$

$$\frac{1}{8} = \frac{1}{11} + \frac{3}{88} \text{ (Does not work)}$$

7. Can all unit fractions be made in more than one way like this? - Yes.

8. Choose different unit fractions of your own to test out your theories.

1.  $\frac{1}{9} = \frac{1}{10} + \frac{1}{90}$
2.  $\frac{1}{9} = \frac{1}{11} + \frac{2}{99}$  (Does not work)
3.  $\frac{1}{9} = \frac{1}{12} + \frac{1}{36}$

Arya observed that Charlie's technique worked only if the difference between the denominators of the first addend and the sum divides their product.

$\frac{1}{9} = \frac{1}{10} + \frac{1}{90}$  works because  $(10 - 9)$  divides  $10 \cdot 9$ . The quotient  $(10 \cdot 9 / 1 = 90)$  is the denominator of the other addend.

$\frac{1}{9} = \frac{1}{12} + \frac{1}{36}$  works because  $(12 - 9)$  divides  $12 \cdot 9$ . The quotient  $(12 \cdot 9 / 3 = 36)$  is the denominator of the other addend.

$\frac{1}{9} = \frac{1}{11} + \frac{2}{99}$  does not work  $(11 - 9)$  does not divide  $11 \cdot 9$

9. Charlie tried to do the same with  $\frac{1}{8}$ . Can you finish Charlie's calculations to see which ones work?

1.  $\frac{1}{8} = \frac{1}{9} + \frac{1}{72}$
2.  $\frac{1}{8} = \frac{1}{10} + \frac{1}{40}$
3.  $\frac{1}{8} = \frac{1}{11} + \frac{3}{88}$  (Does not work)

$\frac{1}{8} = \frac{1}{9} + \frac{1}{72}$  works because  $(9 - 8)$  divides  $9 \cdot 8$ . The quotient  $(9 \cdot 8 / 1 = 72)$  is the denominator of the other addend.

$\frac{1}{8} = \frac{1}{10} + \frac{1}{40}$  works because  $(10 - 8)$  divides  $10 \cdot 8$ . The quotient  $(10 \cdot 8 / 2 = 40)$  is the denominator of the other addend.

$1/8=1/11+3/88$  does not work because  $(11-8)$  does not divide  $11*8$ .

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## USING EQUIVALENT FRACTIONS

Reyansh and Anirved came up with this method. They figured out that if we get the largest unit fraction less than the given unit fraction, we can express it as a sum.

### HOW TO FIND THE LARGEST UNIT FRACTION THAT IS LESS THAN THE GIVEN FRACTION?

**Step 1:** Find equivalent fraction for the given fraction such that you can manipulate the numerator into being a factor of the denominator.

$$23/35 = 46/70$$

**Step 2:** Split the 46 into  $35+11$

$$46/70 = 35/70 + 11/70$$

$35/70$  is nothing but  $1/2$  and that is the largest unit fraction less than  $23/35$

$$\begin{aligned} \text{Therefore } 23/35 &= 1/2 + 11/70 \\ &= 1/2 + 10/70 + 1/70 \\ &= 1/2 + 1/7 + 1/70 \end{aligned}$$

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Anirved came up with all possible solutions for expressing unit fractions as sum of other unit fractions.

**a.**  $1/2=1/3+1/6$

**b.**  $1/3=1/4+1/12$

$$1/3=1/5+1/15+1/15$$

**c.**  $1/4=1/5+1/20$

$$1/4=1/6+1/12$$

$$1/4=1/7+1/14+1/28$$

**h.**  $1/9=1/10+1/90$

$$1/9=1/11+1/99+1/99$$

$$1/9=1/12+1/36$$

$$1/9=1/13+1/30+1/1,170$$

$$1/9=1/14+1/26+1/819$$

$$1/9=1/15+1/45+1/45$$

$$1/9=1/16+1/24+1/144$$

$$1/9=1/17+1/20+1/510+1/3,060$$

**d.**  $\frac{1}{5} = \frac{1}{6} + \frac{1}{30}$

$$\frac{1}{5} = \frac{1}{7} + \frac{1}{35} + \frac{1}{35}$$

$$\frac{1}{5} = \frac{1}{8} + \frac{1}{20} + \frac{1}{40}$$

$$\frac{1}{5} = \frac{1}{9} + \frac{1}{12} + \frac{1}{180}$$

**e.**  $\frac{1}{6} = \frac{1}{7} + \frac{1}{42}$

$$\frac{1}{6} = \frac{1}{8} + \frac{1}{24}$$

$$\frac{1}{6} = \frac{1}{9} + \frac{1}{18}$$

$$\frac{1}{6} = \frac{1}{10} + \frac{1}{15}$$

$$\frac{1}{6} = \frac{1}{11} + \frac{1}{22} + \frac{1}{33}$$

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**i.**  $\frac{1}{10} = \frac{1}{11} + \frac{1}{110}$

$$\frac{1}{10} = \frac{1}{12} + \frac{1}{60}$$

$$\frac{1}{10} = \frac{1}{13} + \frac{1}{65} + \frac{1}{130}$$

$$\frac{1}{10} = \frac{1}{14} + \frac{1}{35}$$

$$\frac{1}{10} = \frac{1}{15} + \frac{1}{30}$$

$$\frac{1}{10} = \frac{1}{16} + \frac{1}{40} + \frac{1}{80}$$

$$\frac{1}{10} = \frac{1}{16} + \frac{1}{32} + \frac{1}{160}$$

$$\frac{1}{10} = \frac{1}{17} + \frac{1}{34} + \frac{1}{85}$$