Part 1

In this investigation, I will be finding out whether Charlie's rules are correct or not, and I will explore the nature of unit fractions and if there is a way to create patterns with them.

Here I solved some of his rules:

Unit fractions (fractions which have numerators of 1) can be written as the sum of two <u>different</u> unit fractions.

For example

$\frac{1}{2} = \frac{1}{3} + \frac{1}{6}$

Charlie thought he'd spotted a rule and made up some more examples.

$\frac{1}{2} = \frac{1}{10} + \frac{1}{20}$ $\quad \frac{1}{2} = \frac{2}{20} + \frac{1}{20} \quad \frac{1}{2} \neq \frac{3}{20}$

$\frac{1}{3} = \frac{1}{4} + \frac{1}{12}$ $\quad \frac{1}{3} = \frac{3}{12} + \frac{1}{12} \quad \frac{1}{3} = \frac{4}{12}$

$\frac{1}{3} = \frac{1}{7} + \frac{1}{21}$ $\quad \frac{1}{3} = \frac{3}{21} + \frac{1}{21} \quad \frac{1}{3} \neq \frac{4}{21}$

$\frac{1}{4} = \frac{1}{5} + \frac{1}{20}$ $\quad \frac{1}{4} = \frac{4}{20} + \frac{1}{20} \quad \frac{1}{4} = \frac{5}{20}$

As we can see, his rules are based off the last denominator being the product of the first 2 denominators (e.g. 3 * 4 = 12). If that's the case, why don't all his rules work? Here are some things I noticed that differs the ones that work from the ones that didn't:

(Before I can explain, I have drawn a diagram for a clearer understanding:)

$$\frac{1}{a} = \frac{1}{b} + \frac{1}{c}$$

- denominator a and b are next to each other on the factor line. Imagine all the factors of a number lined up systematically, from smallest to largest. Denominator a and be would be next to each other.

  (e.g. factors of 12: 1, 2, 3, 4, 6, 12)
- Denominators a and b are in the center of the factor line, and when 2 numbers are in the center of the number line, if either is devised by their common multiple, the result would always be the other.

  (c/b=a and c/a=b.)

- Denominators a and b are adjacent numbers, meaning that they are adjacent to each other on both the number line and the factor line.
- Denominator a is always the smaller number.
- Denominator c is the largest number in the rule, and is also the common multiple of denominator a and b.
- Denominator c is always an even number.

To test if my theories were correct, here are some examples of correct rules:

$$\frac{1}{5} = \frac{1}{6} + \frac{1}{30} \qquad \frac{1}{6} = \frac{1}{7} + \frac{1}{42}$$

$$\frac{1}{5} = \frac{5}{30} + \frac{1}{30} \qquad \frac{1}{6} = \frac{6}{42} + \frac{1}{42}$$

$$\frac{1}{5} = \frac{6}{30} \qquad \frac{1}{6} = \frac{7}{42}$$

Now that we know how to make correct rules, here is the conclusion and my advice to Charlie for making correct rules:

To conclude, in order to make correct unit fraction rules, Charlies must:

1. Choose 2 of any adjacent numbers on the number line.
2. Find the common multiple of the 2 numbers by multiplying them.
3. Write the rule in the same format systematically from the denominator with the smallest value to the largest because the smaller value of a fraction is, the more it is worth.

Part 2

Alison started playing around with $\frac{1}{6}$ and was surprised to find that there wasn't just one way of doing this.

She found:

$\frac{1}{6} = \frac{1}{7} + \frac{1}{42}$

$\frac{1}{6} = \frac{1}{8} + \frac{1}{24}$

$\frac{1}{6} = \frac{1}{9} + \frac{1}{18}$

$\frac{1}{6} = \frac{1}{10} + \frac{1}{15}$

$\frac{1}{6} = \frac{1}{12} + \frac{1}{12}$ (BUT she realised this one didn't count because they were not different.)

Charlie tried to do the same with $\frac{1}{8}$. Can you finish Charlie's calculations to see which ones work?

1 $\frac{1}{8} = \frac{1}{9} + ?$

2 $\frac{1}{8} = \frac{1}{10} + ?$

3 $\frac{1}{8} = \frac{1}{11} + ?$

....

1. let x be the unknown number

$\frac{1}{8} = \frac{1}{9} + x$

$-x = \frac{1}{9} - \frac{1}{8}$

$-x = \frac{8}{72} - \frac{9}{72}$

$-x = \frac{-1}{72} \qquad x = \frac{1}{72}$

2. let y be the unknown number

$\frac{1}{8} = \frac{1}{10} + y$

$-y = \frac{1}{10} - \frac{1}{8}$

$-y = \frac{8}{80} - \frac{10}{80}$

$-y = \frac{-2}{80}$

$y = \frac{2}{80} = \frac{1}{40}$

3. let z be the unknown number

$\frac{1}{8} = \frac{1}{11} + z$

$-z = \frac{1}{11} - \frac{1}{8}$

$-z = \frac{8}{88} - \frac{11}{88}$

$-z = \frac{-3}{88}$

$z = \frac{3}{88}$

I solved these questions with algebra, and my results tell me that the 1st and 2nd rules work, while the third doesn't.

According to my point in Part 1:
Question 1 would definitely work as it's numerators are adjacent numbers on a number line of a common multiple, therefore the rule would work.

Question 2 may seem as though it isn't correct, but it in fact is, because even though 8 and 10 aren't adjacent numbers on the factor and number line, they are if they are divided by 2 (4 and 5).

Question 3 will definitely not work, as 8 and 11 are not adjacent number in any way, therefore this question is completely incorrect.

Can all unit fractions be made in more than one way like this?

Choose different unit fractions of your own to test out your theories.

To conclude, if the rule follows the point mentioned in part 1 and if they are multiplied, the rule would work.

Here is an example of unit fraction working in a similar way:

Let T be the unkown number

$$\frac{1}{4} = \frac{1}{6} + x$$

$$-x = \frac{1}{6} - \frac{1}{4}$$

$$-x = \frac{4}{24} - \frac{6}{24}$$

$$-x = \frac{-2}{24}$$

$$x = \frac{2}{24} = \frac{1}{12} \longrightarrow \frac{1}{3} = \frac{1}{4} + \frac{1}{12}$$

$$\frac{1}{3} = \frac{3}{12} + \frac{1}{12}$$

$$\frac{1}{3} = \frac{4}{12}$$

This shows that not only does this work with all three denominators, it can only work with just 2 denominators, but that does mean that I had to readjust the number’s position in order for it to work.