

What is the sum of the first 30 odd numbers?

Here we need to find a pattern that is available for the current situation. If you noticed, every time they add another odd number to the previous ones, it turns from  $x$  squared to  $(x+1)$  squared. Why? Because when we add another odd number, it covers around the previous odd numbers and expand the two sides each by 1, which is why it turns from  $x$  squared to  $(x+1)$  squared. With this pattern, we can also see that if there is 1 odd number, it's 1 squared. 2 odd numbers, it's 2 squared... so 30 odd numbers, is 30 squared, which is 900.

What is the sum of the first 60 odd numbers?

We can work this out using the method I first explained. But just using bigger numbers. 60 squared is 3600, so the answer is 3600.

Can you describe a method for working these out quickly?

Yes. If you noticed, every time they add another odd number to the previous ones, it turns from  $x$  squared to  $(x+1)$  squared. Why? Because when we add another odd number, it covers around the previous odd numbers and expand the two sides each by 1, which is why it turns from  $x$  squared to  $(x+1)$  squared. With this pattern, we can also see this pattern:

no. of odds  $x^2$

Can you make 3249 by adding odd numbers in this way?

Yes. We can without a calculator:

so 3249 is 57 squared.

$$\begin{array}{r}
 229 \\
 \hline
 7 \times 7 \text{ or } 3 \times 3
 \end{array}
 \quad
 \begin{array}{r}
 5 \\
 \hline
 \sqrt{3249} \\
 25 \phantom{00} \\
 \hline
 \text{(most)}
 \end{array}$$

$$\begin{array}{l}
 57^2 = 3249 \\
 50 + 7 = 57 \\
 50 + 3 = 53 \\
 53^2 = 2809
 \end{array}$$

What is the value of:

$$1+3+\dots+149+151+153?$$

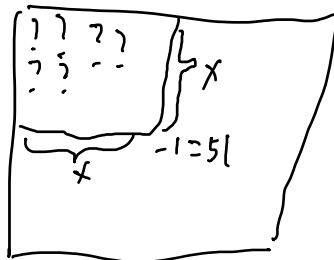
In order to work this out, we already have the pattern. But what is the number we are squaring? If the last number is 153, and it covers the previous numbers completely, then the side length must be  $x$  (length of side) plus  $x-1$ . Because we can imagine 153 as two lines touching each other at the end, but having to minus 1 because of two 1's overlapping. So the answer is  $(153+1)$  divided by 2 together, squared. Which is 5929.

$$83+81+\dots+5+3+1?$$

This question is partially the same, only backwards. We use the same method:  $(83+1)$  divided by 2 together squared, which is 1764.

$$51+53+55+\dots+149+151+153?$$

This question is a bit trickier, for it doesn't start from 1, but 51. So we have to imagine it as a squared number subtracted by a square number.



$$\begin{aligned} (1+5) &= 52 \\ 52 \div 2 &= 26 \quad 26^2 = 676 \end{aligned}$$

$$77^2 = 5929$$

$$5929 - 676 = 5253$$

So the answer is 5253.

$$2+4+\dots+150+152+154?$$

$$\begin{aligned} y & \\ -1 &= 153 \\ -1 &= 153 \end{aligned}$$

In this question, every step is just as the same as the first question we know. Just adds a 1 all together. So the answer is 5930 ( $77$  squared plus 1).

$$2+6+\dots+298+302+306?$$

This is basically the same question, except the fact that the answers here are the original answers multiplied by 2. So the answer is 5929 times 2: 11858.

Explain how you worked these out.

Look up!