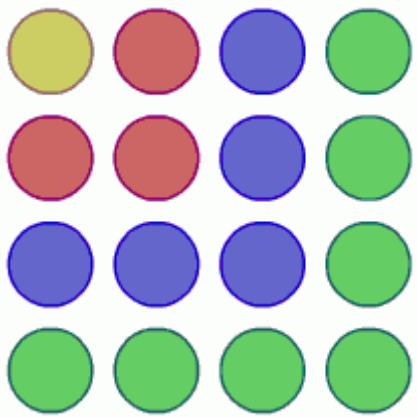


# Picturing Squares Solutions

My solutions on picturing squares :

Pic :



Observations :

- There are different colored circles
- It is a 4 cross 4 square = 16 circles
- The dots go in a sequence of 1,3,5,7 which is a series of odd numbers .
- If you add 2 different colors of dots you get a square . e.g. :  
 $1+3=4$  ,  $4+5=9$ ... etc.
- All the numbers have a gap of 2 .
- If you add two consecutive odd numbers ;  
E.g :  $1+3 =$  now notice that there are two terms .  $4=2$  squared . my observation is that based on the number of terms / odd numbers that will be your square . we saw that  $1+3=4 =2$  square which has 2 terms . So if we want to find out the sum of the first 5 odd numbers then we will get the answer as 5 square . this is true as :  
 $1+3+5+7+9=25 = 5$  square .
- There are 16 circles

- *If you have an odd number of odd numbers you will get an odd square , and if you have an even number of odd numbers you will get an even square .*

$$1+3=4 = 2 \text{ square}$$

*Has an even number of odd numbers . Which gives an even square of 2.*

$$1+3+5=9 = 3 \text{ square}$$

*Has an odd number of odd numbers which is an odd square of 3 .*

*1. What is the sum of the first 30 odd numbers?*

*Answer:*

*So since it is the first 30 odd numbers and as said in the observations above we stated that the number will be the square . so since there are 30 terms we just find the value of 30 square which is :*

$$30 \times 30 = 30 \text{ square} = 900$$

*The sum of the first 30 odd numbers is 900 .*

*Now we can generalize as :*

*no.of terms  $\times$  no. Of terms = sum,*

2. What is the sum of the first 60 odd numbers?

Answer :

So we first have to see the no. of terms like in q.1 . now we can see there are 60 terms as per this current question . so we use our generalized formula as :

No. of terms  $\times$  no. of terms = sum .

$$60 \times 60 = 3600$$

The sum of the first 60 odd numbers is 3600 .

3. Can you describe a method for working these out quickly?

Solution:

As in q.1 and q.2 the method to calculate these in a quick way .(which is also mentioned in the observations ) is :

No. of terms  $\times$  no. of terms = sum

For example sum of the first 56 odd numbers ;

No. of terms = 56

So :

$$56 \times 56 = 56 \text{ square} = 3136$$

Hence the method to calculate quickly is :

No. of terms  $\times$  no. of terms = sum

4. Can you make 3249 by adding odd numbers in this way?

**Solution:**

My first step was just checking for any square numbers in the square number 3249 .

Yes there is a square number 7 .  $7 \times 7 = 49$  this is the last 2 numbers of the number 3249 .

So that means it is an odd square ending in 7 .

The square of 60 = 3600

So  $3249 < 3600$  (60 square )

Then if we try the square 55 :

And the square 55 = 3025 .

$3025$  (55 square )  $< 3249$

So if you try the closest number ending in 7 in 55 square it is 57 .

And the value of 57 square = 3249 .

Hence I conclude that the square of 3249 = 57 square .

5. What is the value of:

a)  $1+3+\dots+149+151+153?$

b)  $83+81+\dots+5+3+1?$

c)  $51+53+55+\dots+149+151+153?$

d)  $2+4+\dots+150+152+154?$

e)  $2+6+\dots+298+302+306?$

Explain how you worked these out.

Solution :

a)  $1+3+\dots+149+151+153=?$

So let us first find the numbers of terms , so we can square the number of terms . This is the same pattern as the previous questions .

First let us find the number of odd numbers between a few numbers to see a pattern .

$$1-10= 1,3,5,7,9 -5 ,10/2=5$$

$$1-17=1,3,5,7,9,11,13,15,17 - 9. 17/2+1.$$

So now for the last odd number we have to divide by 2 then add 1 .

$$153/2=76+1=77$$

And now with formula : no. of terms  $\times$  no.of terms =  $77 \times 77=5929$

The sum of  $1+3+\dots+149+151+153=5929$  .

$$b) 83+81+\dots+5+3+1 = ?$$

**Solution:**

So the formula doesn't change whether it is reversed or not . this is the same method as the previous question a ) . so let us use the same logic as q.a) by :

So as in the previous question we need to find the number of terms and square to get the sum . so we know that if the last number is odd then we divide by 2 then add 1 . so :

$$83/2=41+1=42 .$$

$$\text{So } 42 \text{ square} = 42 \times 42 = 1764$$

c)  $51+53+55+\dots+149+151+153?$

**Solution:**

So let us use some logic in this . so we can see in q.5 a) that the question is to find the sum of  $1+3+\dots+149+151+153$

And we have the same case here except starting from 51 .

The sum of  $1+3+\dots+149+151+153=5929$  .

Now we subtract by the sum of 1-49 . which is simple as the number of odd terms in 1-49 is 25 .(including 49 ) so 25 square which is :

$25 \times 25 = 625$  . Now we subtract .

$5929 - 625 = 5304$  is the sum of  $51+53+55+\dots+149+151+153$

d)  $2+4+\dots+150+152+154?$

**Solution :**

Let's make a table to see some patterns.

seq:	Sum :
$2+4$	6
$2+4+6$	12
$2+4+6+8$	20
$2+4+6+8+10$	30

Continuation: 5. d)

So the general formula :

Last term / 2 x last term / 2 + 1 = sum

e.g. : 2+4

$4/2 \times 4/2 + 1 =$

$2 \times 2 + 1 = 2 \times 3 = 6$  this is the sum

So if we apply the same to 154 , our sum will be :

$154/2 \times 154/2 + 1 = \text{sum}$

$77 \times 77 + 1 =$

$77 \times 78 = 6006$  which is the sum of  $2+4+\dots+150+152+154$  .

e)  $2+6+\dots+298+302+306?$

Solution :

Let's first make a table and find some patterns :

seq:	Sum :
2+6	8
2+6+10	18
2+6+10+14	32
2+6+10+14+18	50

Now we can make general formula that ;

No. of terms x no. of terms x 2 = sum .

Continuation :

Let's use an example  $2+6+10+14=?$

So the no. of terms is 4 and  $4 \times 4 = 16$  and  $16 \times 2 = 32$  and in the table the sum of  $2+6+10+14 = 32$ .

Now we just need to find the number of terms.

$1-10=2,6,10$  (3) terms of even no. with a gap of 4.

$10-20=2$ .

$20-30=3$

So we can see that the pattern is alternative(3,2,3...) and there are 10 groups (as in 1-10, 10-20...) so then in this case we divide  $10 / 2$  (because 2 different number of terms 3,2,3,2,3,2,3,2,3,2) so  $2 \times 5 + 3 \times 5 = 15 + 10 = 25$ . so  $25 \times 3 = 75$ . then add that with 302,306 (extra 2 terms) so  $75 + 2 = 77$ . so 77 terms. so  $77 \times 77 \times 2 = \text{sum}$

Formula : no. of terms  $\times$  no. of terms  $\times 2 = 77 \times 77 \times 2 = 5929 \text{ square} \times 2 = 11858$

The sum of  $2+6+\dots+302+306=11858$

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*(note : i checked my calculation answers with a google calculator to make sure i did not make any calculation errors .)*