

Take three from five

This problem can be solved by using modular arithmetic

If a number a is a multiple of 3, we can write

$$a \equiv 0 \pmod{3}$$

where a is the number

If a number has the remainder of 1, we can write

$$a \equiv 1 \pmod{3}$$

If a number has the remainder of 2, we can write

$$a \equiv 2 \pmod{3}$$

Case 1:

$$n_1 \equiv 0 \pmod{3}$$

$$n_2 \equiv 0 \pmod{3}$$

$$n_3 \equiv 0 \pmod{3}$$

$$n_1 + n_2 + n_3 \equiv 0 \pmod{3}$$

That means that the sum is a multiple of 3.

Case 2

$$n_1 \equiv 1 \pmod{3}$$

$$n_2 \equiv 1 \pmod{3}$$

$$n_3 \equiv 1 \pmod{3}$$

$$n_1 + n_2 + n_3 \equiv 3 \pmod{3}$$

$$\equiv 0 \pmod{3}$$

Also a multiple of 3

Case 3

$$n_1 \equiv 2 \pmod{3}$$

$$n_2 \equiv 2 \pmod{3}$$

$$n_3 \equiv 2 \pmod{3}$$

$$n_1 + n_2 + n_3 \equiv 6 \pmod{3}$$

$$\equiv 0 \pmod{3}$$

Also a multiple of 3

Case 4

$$n_1 \equiv 0 \pmod{3}$$

$$n_2 \equiv 1 \pmod{3}$$

$$n_3 \equiv 2 \pmod{3}$$

$$n_1 + n_2 + n_3 \equiv 3 \pmod{3}$$

$$\equiv 0 \pmod{3}$$

P2/2

Choose 3 numbers from S , sum them up. (Mod 3) The sum of the three congruents are shown here

Congruent

n_1	n_2	n_3	n_4	n_5
0	0	0	0	0
1	1	1	1	1
2	2	2	2	2
0	0	0	0	1
0	0	0	1	1
0	0	1	1	1
0	1	1	1	1
1	1	1	1	2
1	1	1	2	2
1	1	2	2	2
1	2	2	2	2
0	1	2	2	2
0	1	1	2	2
0	1	1	1	2
0	0	1	2	2
0	0	1	1	2
0	0	0	1	2

Sum of 3 numbers $\equiv 0$
 Sum of 3 numbers $\equiv 3 \equiv 0$
 ... $\equiv 6 \equiv 0$
 ... $\equiv 0$

Sum of Congruents $\equiv 0 \pmod{3}$
 i.e a multiple of 3

Legend:

- ✓ The 3 numbers chosen
- ✓ Alternative choices. (In some there are more than two)

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