Stage 4 ★
Mixed Selection 1

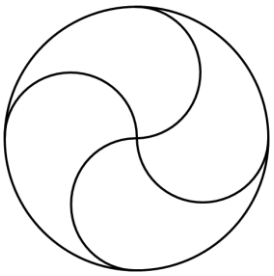
1. Circle in a semicircle

The diagram shows a semicircle containing a circle which touches the circumference of the semicircle and goes through its centre.

What fraction of the semicircle is shaded?

2. Four parts

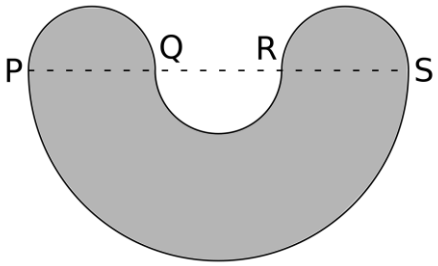
The circle of radius $4cm$ is divided into four congruent parts by arcs of radius $2cm$ as shown.



What is the length of the perimeter of one of the parts, in cm?

3. Smile

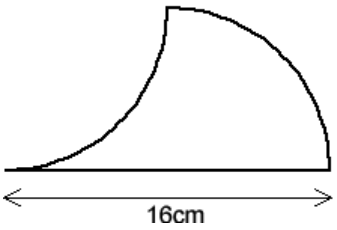
The points P, Q, R, S lie in order along a straight line, with $PQ = QR = RS = 2cm$. Semicircles with diameters PQ, QR, RS and SP join to make the shape shown below.



What is the area of the shape?

4. Arc area

This figure is made from a straight line 16 cm long and two quarter circles, one with its centre at the midpoint of the straight line.



What is the area of the figure (in cm^2)?

These problems are adapted from UKMT Mathematical Challenge problems (ukmt.org.uk)

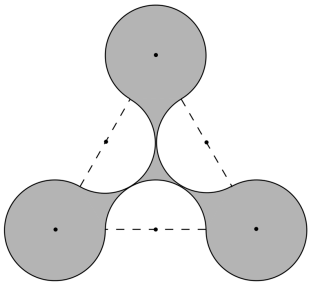
Stage 4 ★
Mixed Selection 2

1. Penny farthing

Boris Biker entered the Tour de Transylvania with an unusual bicycle whose front wheel is larger than the back. The radius of the large wheel is 40cm, and the radius of the small wheel is 30cm. On the first stage of the race the smaller wheel made 120,000 revolutions. How many revolutions did the larger wheel make?

2. Tadpoles

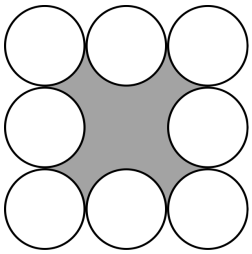
The diagram shows a shaded shape bounded by circular arcs with the same radius. The centres of three arcs are the vertices of an equilateral triangle; the other three centres are the midpoints of the sides of the triangle. The sides of the triangle have length 2.



What is the difference between the area of the shaded shape and the area of the triangle?

3. Square Flower

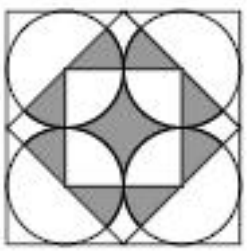
The shaded region is bounded by eight equal circles with centres at the corners and midpoints of the sides of a square.



The perimeter of the square is 8cm. What is the perimeter of the shaded region?

4. Crazy shading

The diagram shows four circles each of which touches the largest square and two adjacent circles. A second square has its vertices at the midpoints of the sides of the largest square and the central square has its vertices at the centres of the circles.



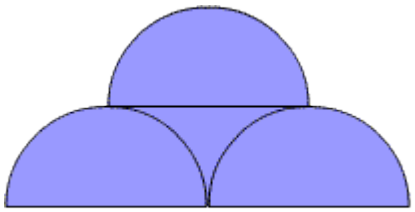
What is the ratio of the total shaded area to the area of the outer square?

These problems are adapted from UKMT Mathematical Challenge problems (ukmt.org.uk)

Stage 4 ★
Mixed Selection 3

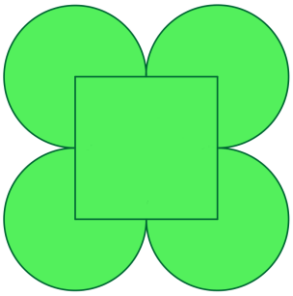
1. Semicircle stack

The diagram shows three semicircles, each of radius one. What is the size of the total area shaded?



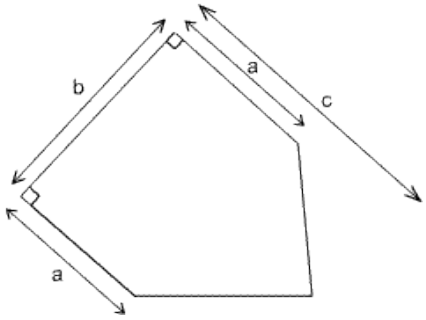
2. Four leaf clover

The diagram shows four equal discs and a square. Each disc touches its two neighbouring discs. Each corner of the square is positioned at the centre of a disc. The side length of the square is $2/\pi$. What is the length of the perimeter of the figure?



3. Pentagonal area

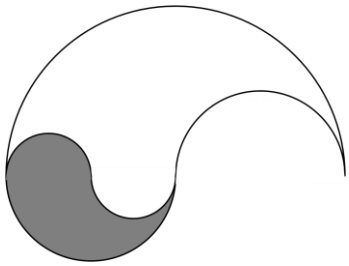
What is the area of the pentagon shown?



4. Semicircular design

The diagram to the right shows a logo made from semi-circular arcs, each with a radius of 2cm, 4cm or 8cm.

What fraction of the logo is shaded?



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