Age 14+ Level ★★★
Worksheet 1

1. **Three Primes**
   How many sets of three prime numbers have the property that the product of the three numbers is exactly five times their sum?
   (The order of the three numbers is not important).

2. **Factorised Factorial**
   For a positive integer n, we define $n!$ to be the product of all the positive integers from 1 to n; that is $n! = 1 \times 2 \times 3 \times \ldots \times n$.

   If $n! = 2^{15} \times 3^{6} \times 5^{3} \times 7^{2} \times 11 \times 13$, what is the value of n?

3. **Factor List**
   When Tina chose a number *N* and wrote down all of its factors, apart from 1 and *N*, she noticed that the largest of the factors in the list was 45 times the smallest factor in the list.

   How many numbers *N* could Tina have chosen for which this is the case?

4. **Primes and Six**
   Let p and q be prime numbers with $q = p + 2$ and p greater than 3.

   Prove that $pq + 1$ is divisible by 36.

5. **Leftovers**
   As n takes each positive integer value in turn (that is, $n = 1$, $n = 2$, $n = 3 \ldots$) how many different values are obtained for the remainder when n^2 is divided by $n + 4$?

6. **Square Product**
   What is the smallest integer n such that the product

   $$(2^2 - 1)(3^2 - 1)(4^2 - 1)\ldots(n^2 - 1)$$

   is a perfect square?

These problems are adapted from UKMT (ukmt.org.uk) and WMC (competition.ac) problems.

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Worksheet 2

1. **Adding a Square to a Cube**
Find all 9 integer values of n between 1 and 100 for which $n^2 + n^3$ is a square number.

2. **Fortunate Inflation**
The price of an item in pounds and pence is increased by 4%.
The new price is exactly n pounds where n is a whole number.

What is the smallest possible value of n?

3. **Rational Integer**
For how many integers n is $\frac{n+3}{n-1}$ also an integer?

4. **Square LCM**
The highest common factor of two positive integers m and n is 12, and their lowest common multiple is a square number.

How many of the five numbers $\frac{n}{3}, \frac{m}{3}, \frac{n}{4}, \frac{m}{4}, mn$ are square numbers?

5. **Cancelling Fractions**
Find a fraction $\frac{m}{n}$ $(m \neq n)$ such that all of the fractions

$$\frac{m}{n}, \frac{m+1}{n+1}, \frac{m+2}{n+2}, \frac{m+3}{n+3}, \frac{m+4}{n+4}, \frac{m+5}{n+5}$$

can be simplified by cancelling.

6. **Super Computer**
Catherine's computer correctly calculates $\frac{66^{66}}{2}$.
What is the units digit of the answer?

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