Stage 3 ⋆
Mixed Selection 1 – Solutions

1. **Right-angled request**

   Four right-angled triangles can be drawn in each of the squares, plus four formed from two adjacent sides of the rectangle - that makes 12.

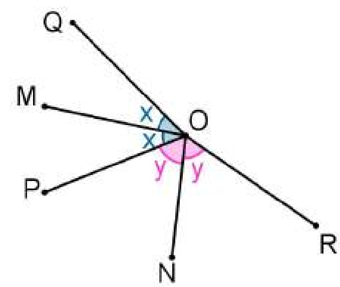
   Then there are two right-angled triangles formed by joining the points UQS and PTR.

   This makes a total of 14 triangles.

2. **Angular reflection**

   As OQ is the reflection of OP in OM, $\angle QOM = \angle POM$.
   Similarly, $\angle RON = \angle PON$.
   Hence, reflex $\angle QOR \quad 2 \times \angle MON = 260°$.

   Therefore, $\angle QOR = 360° - 260° = 100°$.



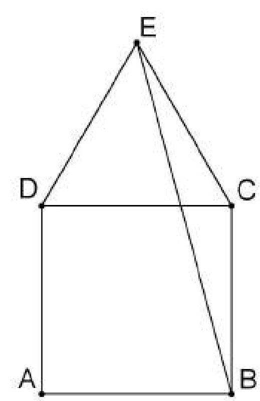
3. **Homely angles**

   Since $ABCD$ is a square, $\angle BCD = 90°$, and since CDE is an equilateral triangle, $\angle DCE = 60°$.

   Thus $\angle BCE = \angle BCD + \angle DCE = 90° + 60° = 150°$.

   Because CDE is an equilateral triangle, $EC = DC$ and also, because $ABCD$ is a square, $DC = CB$. Hence, $EC = CB$ and ECB is an isosceles triangle.

   So $\angle CEB = \angle CBE = \frac{1}{2}(180 - 150)° = 15°$, and hence $\angle BED = \angle CED - \angle CEB = 60° - 15° = 45°$.



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4. Isosceles Meld

Triangle PQR is isosceles. Therefore, $\angle PQR = \angle PRQ = 72°$.
Triangle PSR is also isosceles. Therefore, $\angle RPS = \angle RSP = 36°$.

Therefore, $x = (180 - 36 - 36)° = 108°$.

5. Central distance

Each centre is 2.5 cm from the nearer end. So the distance between them is $\left(9 - (2 \times 2.5)\right)\text{cm} = 4\text{cm}$.

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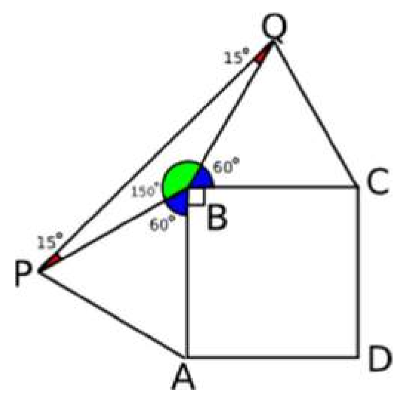
Stage 3 ⋆
Mixed Selection 2 – Solutions

1. **Robo-turn**
The total angle turned through after each of the first 4 moves is $10°, 30°, 60°,$ and $100°$. So the robot does not face due East at the end of a move in its first complete revolution. The total angle it has turned through after each of the next 5 moves is $150°, 210°, 280°, 360°$, and $450°$, so at the end of the 9th move the robot does face East. As the robot moves 5m in each move, the distance it travels is 45m.

2. **Stellar angles**
The four marked angles are the interior angles of a quadrilateral. Hence, $x = 360° - (105° + 115° + 125°) = 15°$.

3. **Two exterior triangles**
Since ABQ and BCQ are equilateral, the angles ABP and CBQ are both 60°. So,
$$\angle PBQ = 360° - 90° - 60° - 60° = 150°$$
PBQ is isosceles, so the angles BPQ and PQB are equal. So,
$$2 \times \angle PQB = 180° - 150° = 30°$$
Therefore, $\angle \mathrm{PQB} = 15°$.



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4. As long as possible

The length of AD must be less than $15cm$, since $15cm$ would be its length if all four points lay in a straight line. However, by making angles ABC and BCD close to $180°$, AD can be made close to $15\ cm$ in length.

As the length of AD is a whole number of centimetres, its maximum value, therefore is 14cm.

5. Polygon cradle

As PQRST is a regular pentagon, each of its internal angles is $108°$. The internal angles of the quadrilateral PRST add up to $360°$ and so, by symmetry, $\angle\text{PRS} = \angle\text{RPT} = \frac{1}{2}(360° - 2 \times 108°) = 72°$. Each interior angle of a regular hexagon is $120°$, so $\angle\text{PRU} = 120°$.

Therefore, $\angle\text{SRU} = \angle\text{PRU} - \angle\text{PRS} = 120° - 72° = 48°$.

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Stage 3 ★
Mixed Selection 3 – Solutions

1. **Parallel base**

Since ST is parallel to UV, $\angle PRT$ and the angle of size $132°$ are corresponding angles, so $\angle PRT = 132°$.

Since angles on a straight line sum to $180°$ we must have $\angle PRQ = 48°$.

By the exterior angle of a triangle theorem, $\angle SQP = \angle QPR + \angle PRQ$, so $134 = x + 48$, that is, $x = 86$.

2. **Other side**

The angles in triangle ACD must add up to $180°$, so $\angle CDA = 180° - 65° - 50° = 65°$.

This means that $\angle ACD = \angle CDA$, so ACD is an isosceles triangle. Therefore, $AC = AD$.

We know from the question that, $AD = BC$, so $BC = AC$. This makes ABC an isosceles triangle, and $\angle CAB = \angle ABC$.

Then $\angle ABC = 12(180° - \angle ACB) = 12(180° - 70°) = 55°$.

3. **Half past two**

In moving from one number on the clock face to the next, a hand moves $360° \div 12° = 30°$.

At $2{:}30$ the hour hand will be exactly half way between the 2 and the 3, and the minute hand will be exactly on the 6. So the angle between the two hands will be $3 \times 30° + 15° = 105°$

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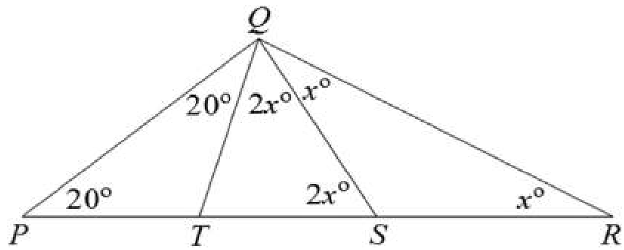
4. Tent poles

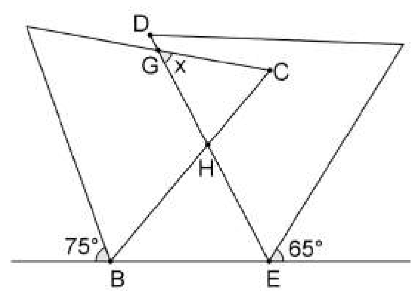
As QS=SR, triangle SQR is isosceles, so $\angle SRQ = \angle SQR = x°$. So by the exterior angle theorem $\angle QST = 2x°$.

Also, $\angle TQS = 2x$ since $QT = TS$.As $PT = QT$, $\angle TPQ = \angle TQP = 20°$.



Since the interior angles of triangle PQR must sum to $180°$, we obtain $20 + (20 + 2x + x) + x = 180$, and hence $x = 35$.

5. Angle of overlap



$\angle CBE = (180 - 75 - 60)° = 45°$
$\angle DEB = (180 - 65 - 60)° = 55°$
$\angle GHB = (45 + 55)° = 100°$ (exterior angle of a triangle)
$\angle HGC = (100 - 60)° = 40°$(exterior angle of a triangle)

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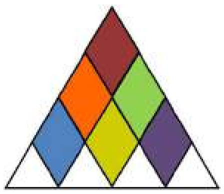
Stage 3 ★
Mixed Selection 4 - Solutions

1. Isometric rhombuses

There are a number of different ways of solving this problem.

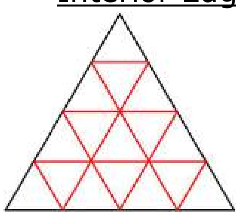
Directions



The number of vertical rhombi can be seen by looking at the possible positions of the top triangle. The following six rhombi are then apparent.
The problem has rotational symmetry, so the other two directions will also give six rhombi each.

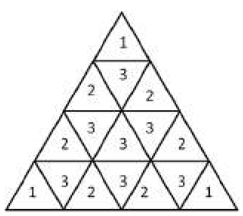
This means that the total number of rhombi is $3 \times 6 = 18$.

Interior Edges



Each rhombus that is formed has exactly one of the interior edges (marked in red) contained within it. Moreover, each interior edge corresponds to one rhombus, consisting of the triangles on either side. There are 18 interior edges, so 18 rhombi that can be formed.

Double-counting



For each of the small triangles, the number of rhombi that contain it can be counted, as shown in the diagram on the right. However, this counts each rhombus twice (once for each triangle it contains). Therefore the total obtained (36) must be halved, giving a total of 18 rhombi.

2. Equilateral pair

Since VXW is an equilateral triangle, $\angle VXW = 60°$. Therefore, $\angle WXY = \angle WXV + \angle VXY = 60° + 80° = 140°$.
Since the equilateral triangles are congruent, $WX = X$, so the triangle WXY is isosceles.
Hence, $\angle YWX = \angle XYW = \frac{1}{2}(180° - \angle WXY) = \frac{1}{2}(180° - 140°) = 20°$.
Then, since VXW is equilateral, $\angle VWX = 60°$.
Then, $\angle VWY = \angle VWX - \angle YWX = 60° - 20° = 40°$.

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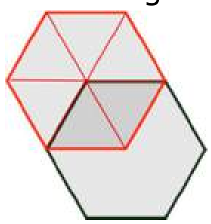
3. Overlapping beer mats

The top beer mat, outlined in red in the diagram below, can be split into triangles from its centre, as shown below.



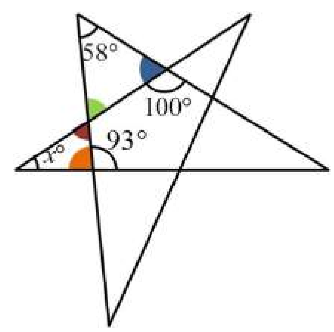
Because the hexagon is regular, the triangles are all the same, so each one will have area $6\ cm^2$ (since the area of the whole beer mat is $36 = 6 \times 6\ cm^2$).

So, since the overlap consists of 2 of the triangles, its area is $12\ cm^2$.

4. Angles please

There are two different ways of calculating the angle x.

The *first method* is as follows:



The blue angle can be calculated since angles on a straight line add to $180°$. This means it is $180° - 100° = 80°$.

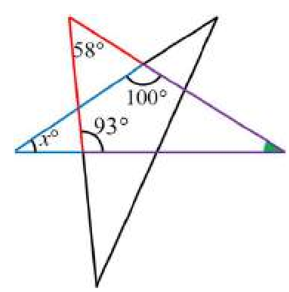
The orange angle can be calculated similarly, to be $180° - 93° = 87°$.

The green angle can then be calculated since the angles in the top-left triangle add up to $180°$. This is $180° - 80° - 58° = 42°$.

The green and red angles are opposite angles. This means that they are equal, so the red angle is also $42°$.

Then, the angles in the left-hand triangle must add up to $180°$, so $x° = 180° - 42° - 87° = 51°$.

The *other method* is by considering the farthest right angle as shown in the diagram.



The angles in the red and purple triangle must add up to $180°$, so the green angle is $180° - 58° - 93° = 29°$.

Then, the angles in the blue triangle must also add up to $180°$.

Therefore, $x° = 180° - 29° - 100° = 51°$.

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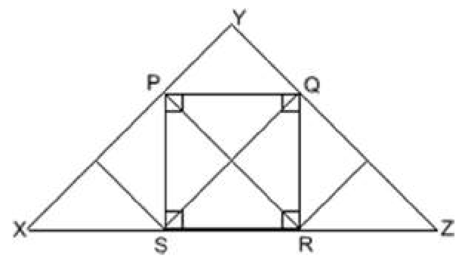
Stage 3 ★★
Mixed Selection 1 –Solutions

1. Square in a triangle

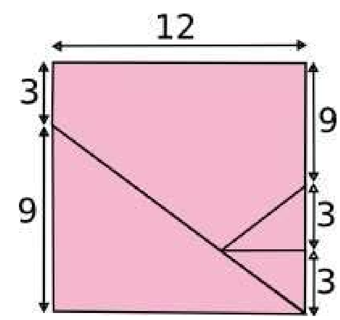
The diagram shows that triangle XYZ may be divided into 9 congruent triangles. The square $PQRS$ is made up of 4 of these 9 triangles. Therefore, the area is $4:9$.



2. Rectangle dissection

The square's perimeter is $12 \times 4 = 48$.

You could also look at the area of the rectangle. This is $16 \times 9 = 144$. This must be the same as the square, so the square must have side lengths of 12. Therefore its perimeter is $12 \times 4 = 48$.



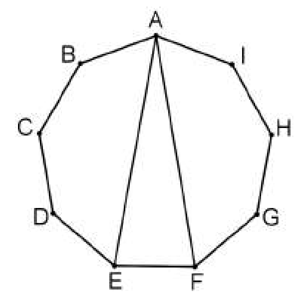
3. Nonagon angle

The interior angle of a regular nine-sided polygon $=$ $180° - (360° \div 9) = 140°$.

Consider the pentagon $ABCDE$:

$$\angle EAB = \frac{1}{2}(540° - 3 \times 140°) = 60°$$

Similarly, $\angle FAI = 60°$ and hence $\angle FAE = 140° - (60° + 60°) = 20°$.

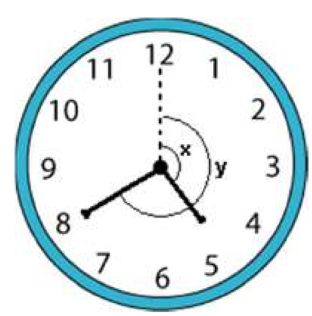


4. Handy angles

Let x denote the angle between the hour hand and the vertical, and y the angle between the minute hand and the vertical.

$\frac{360}{12} = 30$ so we calculate that $x = \left(4 + \frac{2}{3}\right) \times 30 = 140°$ and $y = 8 \times 30 = 240°$.

Hence, the angle between the hands is $y - x = 240° - 140° = 100°$.



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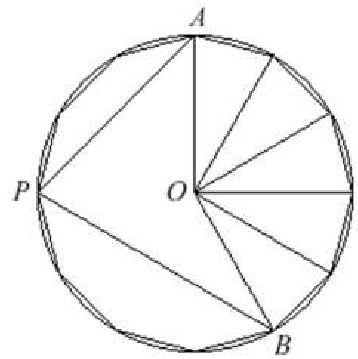
5. Dodecagon angles

Each side of the dodecagon subtends an angle of $30°$ at the centre of the circumcircle of the figure (the circle which passes through all 12 of its vertices).



Since $\angle AOP = 90°$, $\angle OPA = 45°$. Since $\angle BOP = 120°$, $\angle OPB = 30°$. Therefore, $\angle APB = 45° + 30° = 75°$.

Alternatively, $\angle AOB = 150°$ and, as the angle subtended by an arc at the centre of a circle is twice the angle subtended by that arc at a point on the circumference, $\angle APB = 75°$.

6. Outside the boxes

At each vertex of the triangle, four angles meet which must add up to $360°$. Therefore, altogether the twelve angles add up to $3\ x\ 360°$. Six of the angles are right-angles, and three of them are the internal angles of the triangle, which add up to $180°$.

So the sum of the remaining three angles must be

$$(3 \times 360°) - (6 \times 90°) - 180° = 360°.$$

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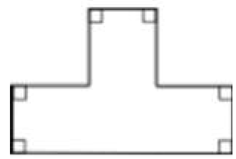
Stage 3 ★★
Mixed Selection 2 – Solutions

1. Right angled octagon

6 right angles can be achieved, as in the diagram below:



If there were seven right angles, these would be a total of $7 \times 90° = 630°$. The total interior angle of an octagon is $6 \times 180° = 1080°$, so the final angle would have to be $1080° - 630° = 450°$. The interior angle cannot be more than $360°$, so this cannot be achieved.

2. Fangs

Alternate angles BDF and DFG are equal, so lines BD and FG are parallel.
Therefore, $\angle BCA = \angle FGC = 80°$ (corresponding angles).

Consider triangle ABC: $x + 70 + 80 = 180$, so $x = 30°$.

3. Integral polygons

The greatest number of sides the polygon could have is 360.

As each interior angle of the polygon is a whole number of degrees, the same must apply to each exterior angle. The sum of the exterior angles of a polygon is $360°$ and so the greatest number of sides will be that of 360-sided polygon in which each interior angle is $179°$, thus making each exterior angle $1°$.

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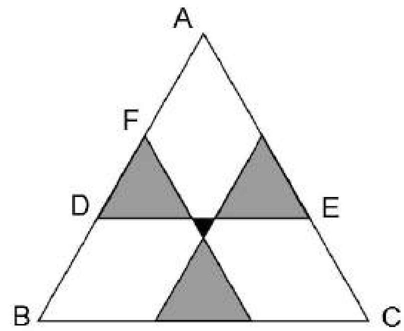
4. Radioactive triangle

As triangle ABC is equilateral, $\angle BAC = 60°$.
Since the grey triangles are equilateral, $\angle ADE = 60°$, so the triangle ADE is equilateral.



The length of the side of this triangle is equal to the length of $DE = (5 + 2 + 5)\text{cm} = 12\text{cm}$.
So $AF = AD - FD = (12 - 5)\text{cm} = 7\text{cm}$.

By a similar argument, we deduce that $BD = 7\text{cm}$, so the length of the side of the triangle $ABC = (7 + 5 + 7)\text{cm} = 19\text{cm}$.

5. Rhombus diagonal

Adjacent angles on a straight line add up to $180°$, so $\angle GJF = 180° - 111° = 69°$.
In triangle FGJ, $GJ = GF$ so $\angle GFJ = \angle GJF$.
Therefore, $\angle FGJ = (180 - 2 \times 69)° = 42°$.
Since $FGHI$ is a rhombus, $FG = FI$ and hence $\angle GIF = \angle FGI = 42°$.

Finally, from triangle FJI, $\angle JFI = (180 - 111 - 42)° = 27°$.

6. Inscribed hexagon

As the sum of the angles in a triangle is $180°$ and all four angles in a rectangle are $90°$, then sum of the two marked angles in the triangle $180° - 90° = 90°$.
Each interior angle of a regular hexagon is $120°$ and the sum of the angles in a quadrilateral is $360°$; hence the sum of the two marked angles in the quadrilateral is $360° - 90° - (360° - 120°) = 30°$.
Hence the sum of the four marked angles is $90° + 30° = 120°$.

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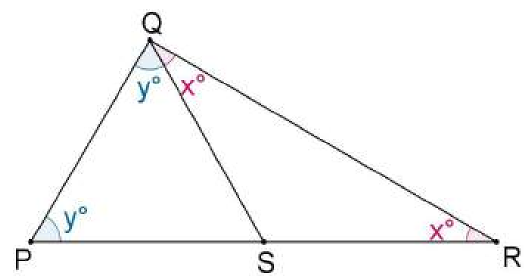
Stage 3 ★★
Mixed Selection 3 – Solutions

1. **Triangle split**
Since $SP = SQ$, the triangle PSQ is isosceles. Therefore, $\angle SPQ = \angle PQS$. We denote the measure of those angles by y.
Similarly, $\angle SQR = \angle QRS = x°$.



Since the sum of the interior angles of PQR is $180°$, $x + y + (x + y) = 180°$, so $2x + 2y = 180°$.

Therefore, $\angle PQR = x° + y° = 90°$.

2. **Hexapentagon**
Each interior angle of a regular pentagon is $108°$, whilst each interior angle of a regular hexagon is $120°$. The non-regular pentagon in the centre of the diagram contains two angles which are interior angles of the regular hexagon, two angles which are interior angles of the regular heptagon and a fifth angle, the one marked x.

So: $x + 2 \times 120 + 2 \times 108 = 5 \times 108 = 540$. Hence $x = 84$.

3. **Extended parallelogram**
Opposite angles of a parallelogram are equal, so $\angle QPS = 50°$.
Therefore, $\angle QPT = 112°$ and, as triangle QPT is isosceles, $\angle PQT = (180° - 112°)/2 = 34°$.
As $PQRS$ is a parallelogram, $\angle PQR = 180° - 50° = 130°$.

So, $\angle TQR = 130° - 34° = 96°$.

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4. Six minutes past eight

At 8 o'clock, the obtuse angle between the hands of the clock is $120°$. In the following six minutes, the minute hand turns through an angle of $36°$ whilst the hour hand turns through an angle of $3°$ in the same direction (clockwise!). So the obtuse angle between the hands increases by $33°$, to $153°$.

5. U in a pentagon

Each interior angle of a regular pentagon is $108°$, so $\angle SRQ = 108°$. As $SR = QR$, the triangle is isosceles with $\angle RQS = \angle RSQ = 36°$. Similarly, $\angle SRT = \angle STR = 36°$. So $\angle SUR = (180 - 2 \times 36)° = 108°$. From the symmetry of the figure,

$$\angle PUR = \angle PUS = (360° - 108°)/2 = 126°.$$

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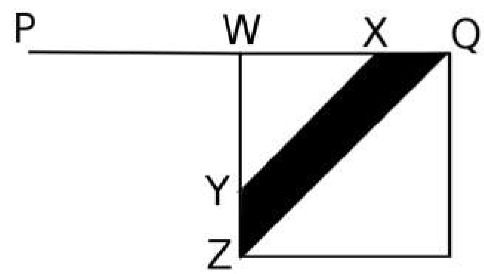
Stage 4 ★★
Mixed Selection 1 - Solutions

1. **Quarters**

The diagram shows the top-right-hand portion of the square.



The shaded trapezium is labelled QXYZ and W is the point at which ZY produced meets PQ. As QXYZ is an isosceles trapezium, ∠QZY = ∠ZQX = 45°.

Also, as YX is parallel to ZQ, ∠XYW = ∠WXY = 45°. So WYX and WZQ are both isosceles right-angled triangles. As ∠ZWQ = 90° and Z is at centre of square PQRS, we deduce that W is the midpoint of PQ. Hence WX = XQ = PQ/4. So the ratio of the side-lengths of similar triangles WYX and WZQ is 1:2 and hence the ratio of their areas 1:4.

Therefore, the area of trapezium QXYZ = $\frac{3}{4}$ x area of triangle ZWQ = $\frac{3}{32}$ x area PQRS since triangle ZWQ is one-eighth of PQRS. So the fraction of the square which is shaded is 4 x $\frac{3}{32}$ = $\frac{3}{8}$.

2. **Angle to chord**

Let O be the centre of the circle. Then $\angle POR = 90°$ as the angle subtended by an arc at the centre of a circle is twice the angle subtended by that arc at a point on the circumference of the circle.
So triangle POR is an isosceles right-angled triangle with $PO = RO = 4\text{cm}$. Let the length of PR be x cm.
Then, by Pythagoras' Theorem, $x^2 = 4^2 + 4^2 = 2 \times 4^2$ and so $x = 4\sqrt{2}$

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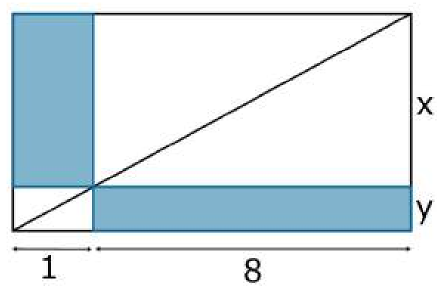
3. Diagonal touch

Let x and y be the distances shown.



Then the shaded area is $8y + x$. But there are a number of similar triangles and from one pair
$\frac{x}{8} = \frac{y}{1}$ therefore $x = 8y$.

So,

$$\frac{shade \quad area}{total\, area} = \frac{8y + x}{9(x + y)} = \frac{8y + 8y}{9} \times 9y = \frac{16}{81}$$

4. Isosceles reduction

Triangles PRS and QPR are similar because $\angle PSR = \angle QRP$ (since $PR = PS$) and $\angle PRS = \angle QPR$ (since $QP = Q$).

Hence $\frac{SR}{RP} = \frac{RP}{PQ}$, that is $\frac{SR}{6} = \frac{6}{9}$, that is $SR = 4$.

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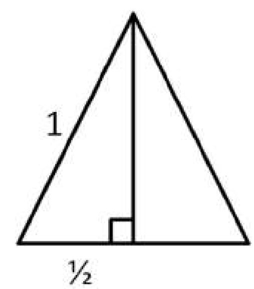
Stage 4 ★★
Mixed Selection 2 – Solutions

1. Two right angles

Triangles QPR and RPS are similar. So: $\frac{PR}{PS} = \frac{PQ}{PR}$. Hence $PR^2 = PQ \times PS = \frac{7}{3} \times \frac{48}{7} = 16$. So PR is 4 units long.

2. Internal – external

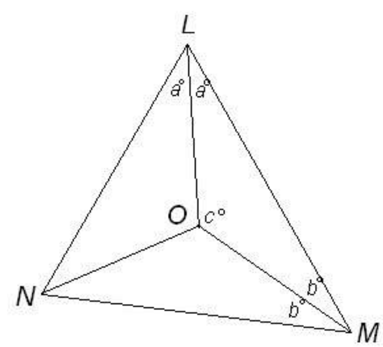


$\angle PSR = 90°$, as it is in a square.
$\angle USR = \angle TSP = 60°$, as they are in equilateral triangles.

Then, $\angle PSU = 90° - 60° = 30°$.
$\angle TSU = 60° + 30° = 90°$.

Thus, TSU is a right angled triangle, so using Pythagoras' theorem: $UT = \sqrt{1^2 + 1^2} = \sqrt{2}$.

3. Incentre angle

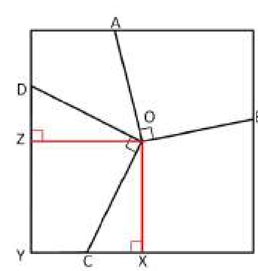


Let $\angle OLM = \angle OLN = a°, \angle OML = \angle OMN = b°$ and $\angle LOM = c°$.

Angles in a triangle add up to $180°$, so from $\Delta LMN, 2a° + 2b° + 68° = 180°$, which gives $2(a° + b°) = 112°$. In other words $a° + b° = 56°$.

Also, from $\Delta LOM, a° + b° + c° = 180°$, and so $c° = 180° - (a° + b°) = 180° - 56° = 124°$.

4. Shaded square



Consider connecting O to X and Z, both of which are the midpoints of the sides. Then, $OX = OZ$ as O is the centre of the square, and $\angle OXC = 90° = \angle OZD$.

Then, $\angle DOC = 90° = \angle ZOX$, so:
$\angle DOZ = 90° - \angle ZOC = \angle COX$. This makes DOZ and COX congruent, as they have the same angles and one pair of corresponding sides the same length.

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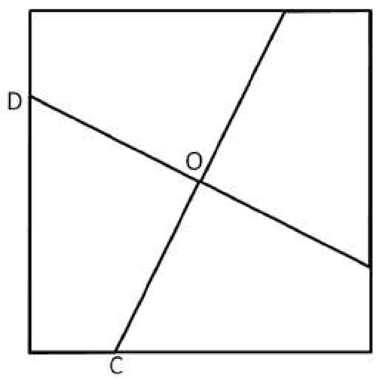
But then, $CODY$ and $ZOXY$ have the same areas, as they both attach one of these triangles to $COZY$. This is clearly a quarter of the large square.

There are two of these unshaded areas, and both have area one quarter that of the square. This means half the square is shaded, so $12 \times 22 = 2\mathrm{m}^2$.

Alternatively, extending the lines from D and C past O divides the square into four congruent pieces, since rotation by $90°$ takes each piece to the next. Therefore, each unshaded area is one quarter of the total area of the square.



There are two of these unshaded areas, and both have area one quarter that of the square. This means half the square is shaded, so $12 \times 22 = 2\mathrm{m}^2$.

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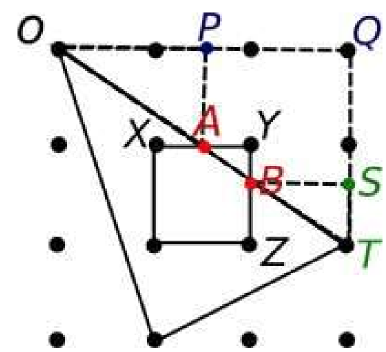
Stage 4 ★★★
Mixed Selection 1 - Solutions

1. Griddy region

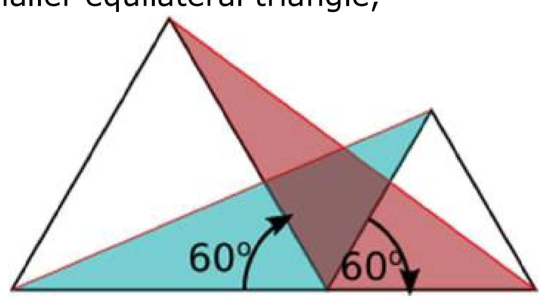
We want to find out how far the points A and B (the points on both the triangle and square) are from Y. The triangle OQT is 3 units across by 2 units down. The triangle OPA is similar to OQT, and is half its size (since it is one unit down rather than two). So the point A is 3/2 units from O, so 3/2 - 1 = 1/2 units from X, so 1 − 1/2 = 1/2 unit from Y.



Similarly, the triangle BST is 1/3 the size of OQT, so the point B is 2/3 units from Z, so 1 - 2/3 = 1/3 unit from Y. So the triangle AYB has area 1/2 x 1/2 x 1/3 = 1/12 square units, and the area of overlap is 1 − 1/2 = 11/12 square units.

2. Two equilateral triangles

Consider the coloured triangles in the diagram below. They both have one side equal in length to the larger equilateral triangle, and these sides are at an angle of 60° to each other. They also both have one side equal in length to the smaller equilateral triangle, and these sides are also at an angle of 60° to each other.



So a 60° rotation will map the blue triangle onto the red triangle. So the two triangles must be congruent. So the third sides, which are the red lengths, must be equal.

A fuller solution is available at: https://nrich.maths.org/12950/solution

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3. Cut-up square

First, we label the corners of the square, the midpoint, and the point at which the two lines intersect A, B, C, D, E and F respectively. We also label the four triangles created within the square as 1, 2, 3 and 4.

Call the area of triangle a.

Angle AFD is equal to angle EFC as they are vertically opposite, and angle FAD is equal to angle FCD as they are alternate. So, triangles 1 and 3 are similar. As E marks the midpoint of a side of the square, length CE is half the value of length AD.

This means that the area of triangle 3 is equal to 4a, as its base and height are double that of triangle 1.

It also means that length AF is equal to twice the value of length FC. If we let AF be the base of triangle 3 and FC be the base of triangle 2, then these triangles have the same height (equal to the perpendicular distance from D to the line AF) so the area of triangle 2 is equal to half that of triangle 3 - that is, 2a.

As line AC is a diagonal, the combined areas of triangles 2 and 3 is equal to half the area of the square. 4a + 2a = 6a, so the square has a total area of 12a. The combined area of triangle 1 and triangle 4 is then also 6a. As we have defined the area of triangle 1 as a, we then know that triangle 4 has an area of 5a.

As the ratio of P: Q is simply the ratio area of triangle 2:area of triangle 4, we then know that this ratio is 2a:5a=2:5.

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4. Garden fence

In the diagram below, triangle ABC represents the garden, CD represents the fence and E is the foot of the perpendicular from D to AC.

The two sections of the garden have the same perimeter so AD is 10m longer than DB. Hence, $AD = 30\text{m}$ and $DB = 20\text{m}$.

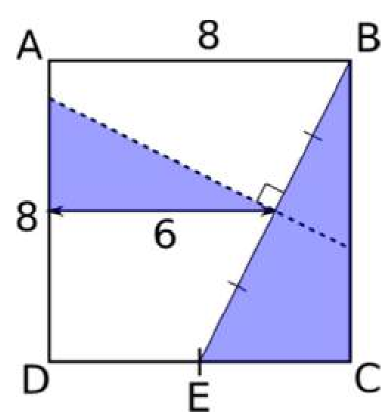
Triangles AED and ACB are similar, so $\frac{AE}{AC} = \frac{AD}{AB} = \frac{30}{50}$. Hence, $AE = \frac{3}{5} \times 30\text{m} = 18\text{m}$.

Also, $\frac{ED}{CB} = \frac{AD}{AB} = \frac{30}{50}$. Hence, $ED = \frac{3}{5} \times 40\text{m} = 12\text{m}$.

Finally, by Pythagoras: $CD^2 = EC^2 + ED^2 = (12^2 + 24^2)\text{m}^2 = 5 \times 12^2\text{m}^2$. So the length of the fence is $12\sqrt{5}\text{m}$.

5. Folded square

In the diagram below, the line EB has been added. The dotted line must be the perpendicular bisector of EB, since for B to fold onto E, B and E must be the same distance on either side of the fold.



The two triangles coloured blue are similar, as they both contain a right angle and their other two angles are 90° rotations of each other.

The dotted line and the folded line cross halfway between E and B (since EB is bisected), so 4 cm from the top (and from the bottom) and 2 cm from the right (and from E). So the distance marked on the diagram below is 6 cm.

The sides of the larger triangle are in the ratio 1:2 (from 4:8), so the vertical side of the smaller triangle must be 3 cm. So the dotted line meets AD 4−3 = 1 cm below A.

A fuller solution is available at: https://nrich.maths.org/12983/solution

These problems are adapted from UKMT Mathematical Challenge problems (ukmt.org.uk)

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