Stage 3 ★
Mixed Selection 1 - Solutions

1. Net result

When the shape is folded up, imagine *X* is on top. This means *B* is on the left hand face and *C* is on the right hand face.

This means *A* is on the back face and *D* is on the front face. This leaves *E* on the bottom face, so opposite *X*.

2. Cubic vision

We can see the greatest number of cubes when looking at three faces at once.

We count the cubes on each face, giving $3 \times 11^2 = 363$ cubes, but we have to subtract from this the cubes along the three edges that have been counted twice, and then add back for the cube at the corner for which three faces are visible.

The final quantity is $363 - (3 \times 11) + 1 = 331$ cubes.

3. Dicey directions

The table shows the number of the face in contact with the table at the various stages described, and also the numbers on the three faces of the die that are visible from the viewpoint in the question.

	Start	1st move	2nd move	3rd move	4th move
In contact with the table	5	1	2	3	5
Facing South	3	3	3	5	4
Facing East	1	2	6	6	6
Facing Up	2	6	5	4	2

As the '6' face is now facing East, the '1' face will be facing West.

4. Painted purple

The solution depends on how the cube is painted.

If you paint the large cube so that two of the small corner cubes have their three painted faces all painted the same colour, then 12 of the small cubes will have at least one red face and also at least one blue face.

If you paint the large cube so that no small corner cube has all three of its painted faces painted the same colour, then 16 small cubes will have at least one red face and one blue face.

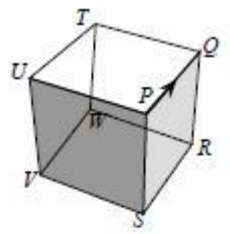
5. Sugary diversion

The ant climbs up the cube (adding 1cm), walks across the top of the cube on the same path as he would have done if the cube wasn't there, then climbs back down to the table (adding another 1cm), so the total extra distance is 2cm.

These problems are adapted from UKMT Mathematical Challenge problems (ukmt.org.uk).

Stage 3 ★
Mixed Selection 2 - Solutions

1. **Crawl around the cube**
   At Q the ant can choose first to go left to T, then right to W. Otherwise, at Q it can go right to R and then left to W.
   W is the corner diagonally opposite to P and is reached by either route after three edges (and no fewer). So after exactly three more edges, the ant must reach the corner opposite W, that is, P.



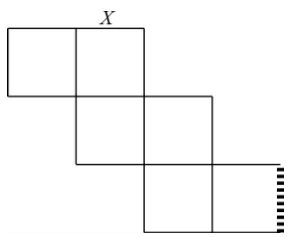
2. **Painted octahedron**
   Two colours is enough: pick any triangle, and colour it red or blue (say, red). Then its three neighbouring faces must be blue. For each of the blue faces, each of its neighbours has neighbours which are either empty or blue – so we may paint them all red. These new red faces all share one empty neighbour, which we may colour blue.

3. **Daniel's star**
   Each pyramid has five faces, but the square base is glued onto the cube, and so is not a face of the star. There are 6 pyramids, each contributing 4 faces, so the star has 24 faces.

   There is one height of pyramid which will make the two faces that meet at each of the original edges become one face, giving a solid with 12 rhomboidal faces.

4. **Four cubes**
   Each cube has 6 identical faces, which have area $24 \div 6 = 4\text{cm}^2$
   The cuboid has 16 of these same square faces, and thus has area $16 \times 4\text{cm}^2 = 64\text{cm}^2$

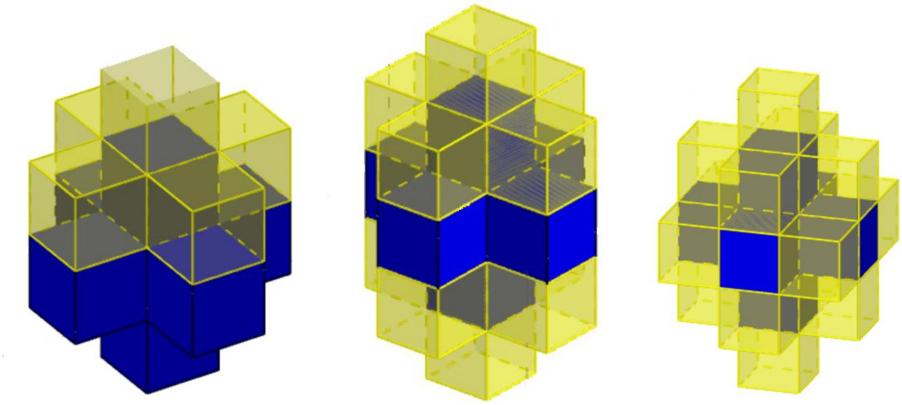
5. **Net profit**
   The dashed edge in the diagram meets X when the net is folded into a cube.



These problems are adapted from UKMT Mathematical Challenge problems (ukmt.org.uk).

Stage 3 ★★
Mixed Selection 1 - Solutions

1. **Cubic covering**
   We glue the yellow faces to the up and bottom faces of the blue cross. We require 5 yellow cubes for wrapping each of the two blue cubes. (We used transparent yellow cubes)

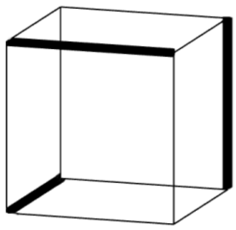
   Now we need one yellow cube for each of the four corners

   Finally, we have 4 faces to cover, so we need 4 more cubes.

   Therefore, we used 5+5+4+4=18 yellow cubes.

2. **Red or black**
   Each edge of the cube borders two faces. As there are six faces, a minimum of three black edges will be required.

   The diagram shows that three black edges is enough.

3. **Truncated tetrahedron**
   At each vertex, the piece that has been removed is a regular tetrahedron. Three of the edges of this are the pieces of edge that are lost from the original tetrahedron, the other three are the new edges of the truncated tetrahedron. Since all the edges are the same length, this means the total length of the edges remains unchanged.

   Therefore, the total length of the edges is 6cm×6=36cm.

These problems are adapted from UKMT Mathematical Challenge problems (ukmt.org.uk).

4. Facial sums

Let the numbers at two of the other vertices be *u* and *v.* The three faces sharing the vertex labelled with the number 1 all have the same sum. Then 1+*v*+*u*=1+5+*u* and so *v*=5.

Similarly, 1+*v*+5=1+*v*+*u* so *u*=5.

Hence the sum for each faces is 1+5+5=11, and we see that the number at the bottom vertex is 1.

The total of all the vertices is 1+5+5+5+1=17.

5. Pyramidal n-gon

The base consists of 1 face and n edges. At each of the n vertices of the base, there is a unique edge which passes through the peak of the pyramid. Each edge corresponds to a unique face (say, the one on its left). This gives n+1 faces and 2n faces – giving a difference of: $2n - (n + 1) = n - 1$