Negative powers

Take a look at this expression. What does it mean?

$$((-4^{-3})^{-2})^{-1}$$

Do you think it means

$(((-4)^{-3})^{-2})^{-1}$ (Interpretation A) or $((-(4^{-3}))^{-2})^{-1}$ (Interpretation B)?

Does it make a difference how we interpret $((-4^{-3})^{-2})^{-1}$?

Do we still get the same value if we change the order of 1, 2 and 3 in the powers?

Now think about changing the order of 1, 2, 3 and 4 in $((-4^{-3})^{-2})^{-1}$.

Which order of 1, 2, 3 and 4 makes the highest value?

Which order makes the lowest value?

How many different values can you get?

Check your thinking:

By usual mathematical conventions for orders of operations, $((-4^{-3})^{-2})^{-1}$ means $((-(4^{-3}))^{-2})^{-1}$ (Interpretation B). You can include the extra brackets if you find these helpful, but make sure you put them in the right place.

Both $(((-4)^{-3})^{-2})^{-1}$ and $((-(4^{-3}))^{-2})^{-1}$ have the same value, so in this case it doesn't make a difference.

However, if we swap the order of 3 and 2, then Interpretation A gives us

$$(((-4)^{-2})^{-3})^{-1} = \left(\left(\frac{1}{(-4)^2}\right)^{-3}\right)^{-1} = \left(\frac{1}{16}\right)^{(-3)\times(-1)} = \left(\frac{1}{16}\right)^{3} = \frac{1}{16^3}$$

But Interpretation B gives us

$$\left(\left(-(4^{-2})\right)^{-3}\right)^{-1} = \left(\left(-\frac{1}{4^2}\right)^{-3}\right)^{-1} = \left(-\frac{1}{4^2}\right)^{(-3)\times(-1)} = \left(-\frac{1}{16}\right)^{3} = -\frac{1}{16^3}$$

So, in this case, it does matter how we interpret $((-4^{-2})^{-3})^{-1}$.