Product Sudoku

First, we assumed that the Sudoku was a coordinate system and each grind in the Sudoku can be shown with coordinates such as $(1,2)$. Thus, each number had its own coordinate for example, the number in the first row and first column is $(1,1)$.

Then, we prime factorized the numbers:

$20 = 2^2 \times 5$ $30 = 2 \times 3 \times 5$ $405 = 3^4 \times 5$ $180 = 2^2 \times 3^2 \times 5$

$96 = 2^5 \times 3$ $2835 = 3^4 \times 5 \times 7$ $4 = 2^2$ $600 = 2^3 \times 3 \times 5^2$

$324 = 2^2 \times 3^4$ $36 = 2^2 \times 3^2$

As we can know from the instructions, $(1,8)$ is 4 and (2,9) is 5, therefore $(1,2)$ cannot be 4 and 2 because $(2,1)$ will need to be 2 too, hence block $(2,1)$ is 4 and block $(1,2)$ is 1.

Knowing that block $(9,7) \cdot (8,6) \cdot (9,5) = 3$ we can get the three blocks have the numbers "1", "1", and "3" from prime factorizing 3, then we can know from the essence of Sudoku that $(8,6)$ and $(9,7)$ is 1, and $(9,5)$ is 3.

By factorizing 600 we can get the four numbers around $(5,3)$ can be $5,5,3,8$ or $5,5,4,6$. These two groups of numbers both contain two 5s so we can know that they are placed on $(5,4)$ and $(4,3)$ or $(5,4)$ and $(6,3)$, and then $(5,4)$ will be confirmed as 5.

From the factorization of 2835, we can know that the four blocks around $(3,2)$ need to be $9,9,7$, and 5. We can know that $(2,2)$ cannot be 5 from the 5 on square $(2,9)$ and can't be on square on $(3,3)$ by the 5 on either $(4,3)$ and $(6,3)$. 9 and 9 are the same number, so they are placed on $(3,3)$ and $(4,2)$. When we already have both nines and a five, we must put 7 on the square $(2,2)$

By factorizing 30, we can get that there needs to be an 5 around $(6,9)$, and from $(2,9)$ we can get that $(6,9)$ and $(7,9)$ cannot be 5, so $(6,8)$ needs to be 5. The 5 on square $(6,8)$ forbids 5 to be on $(6,3)$ making 5 on the square $(4,3)$.

Knowing that 405 has one 5 as its factor, we can easily work out that the 5 needs to be placed on square $(7,6)$. 180 also has one 5 as its factor which is placed on $(1,5)$ because it cannot be placed on $(2,4)$ nor can it be placed on $(1,3)$ because of the two 5s on $(5,4)$ and $(4,3)$. The rest two squares can only be either 6 and 6 or 9 and 4. However, the 4 on $(1,8)$ and the 9 on $(3,3)$ forbid the second option therefore square $(2,4)$ and $(1,3)$ are 6.

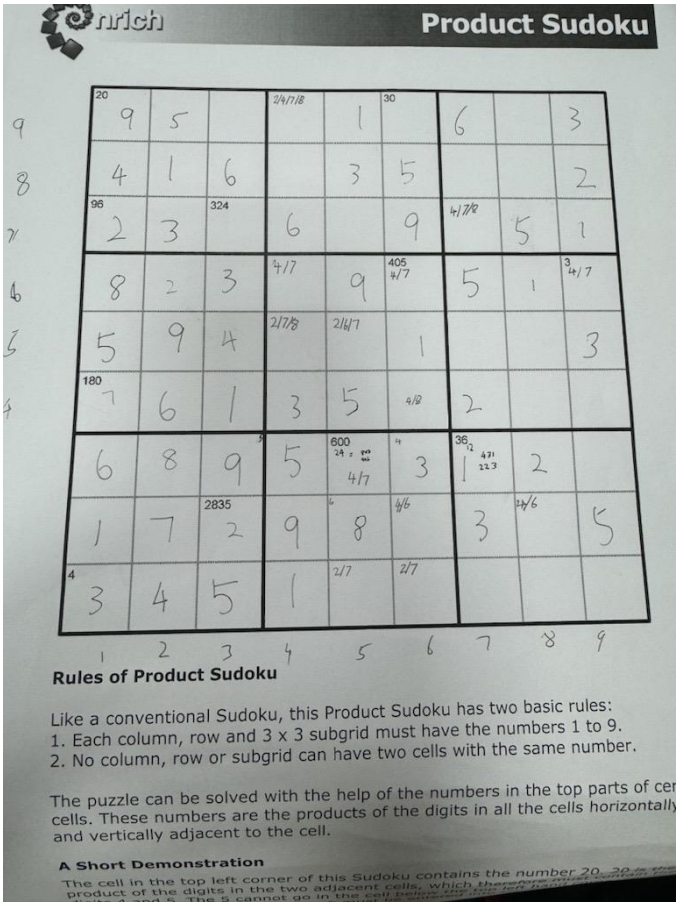
We then shift our attention to the 600 on square $(5,3)$, there are two 5s on the top and side of it, when we divide 600 by 25, we get 24. The two sets of factors of 24 are $3,8$ and $4,6$. The option of 4 and 6 doesn't work because the square $(6,3)$ must be 4 and the rest three square of 36 needed to be $3,3,1$ nor can they be $9,1,1$. However, 1 cannot be in any of

these squares because the two 1s on $(8,6)$ and the 1 on $(1,2)$. The three squares to the sides of 36 can be either $4,3,1$ or $2,2,3$and $6,2,1$. Option 1 is also not possible because as said before, 1 cannot be there so it must be $2,2,3$, therefore $(7,2)$ is 3, $(7,4)$ and $(8,3)$ is 2.

When we already have 2 and 3 on column 7 hence the two squares to the either side of $(6,9)$ cannot be 2 or 3, it can only be 6 or 1. There is already a 1 on the square $(9,7)$ so the 1 is on the left of $(6,9)$ and 6 is on the right.

Because there is already a 5 on $(7,6)$, so the other three blocks around $(6,6)$ need to be $9,9,1$ or $3,3,9$ ($405 \div 5 = 81$). They must be $9,9,1$ because $(6,3)$ and $(9,5)$ is 3, therefore $(5,6)$ and $(6,7)$ is 9, $(6,5)$ is 1.

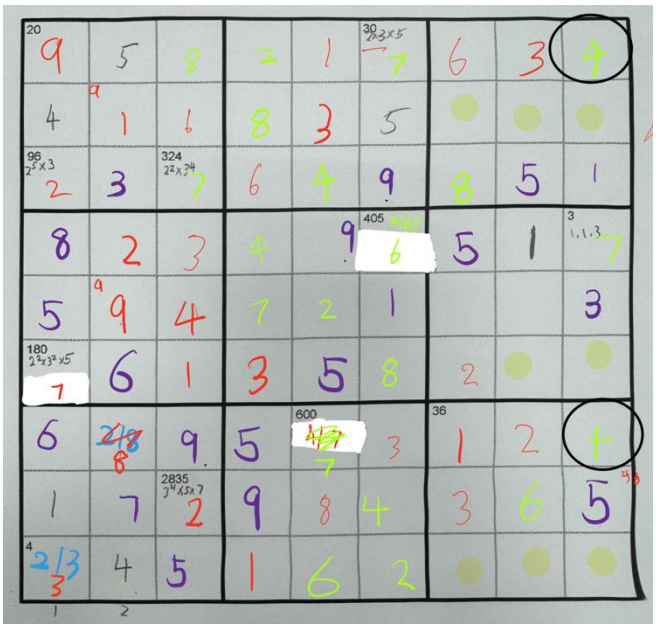
The two squares to the left and bottom of $(1,7)$ could be $3,8$ or $4,6$. Because there is a 4 on $(1,8)$ the set of numbers can only be 3 and 8. As we already know, 324 cannot be divided by 8 hence the number on $(2,7)$ is 3 and $(1,7)$ is 8. The four squares around $(3,7)$ need to be $4,9,3,3$ or $2,6,3,9$ or $3,3,6,6$ or $9,3,2,6$ or $1,6,6,9$. The only available set of numbers remaining is $3,3,6,6$ because $(5,6)$, $(4,2)$, $(3,3)$ are all 9, so $(3,6)$ is 3, $(4,7)$ and $(3,8)$ are 6.
From this point on we have filled in all the numbers that are related to the additional rule. Turning this into regular Sudoku.
First, we analyzed where all the 1s could be, seeing that we have a lot of 1s. Square $(1,8)$, $(3,4)$, $(7,3)$, $(4,1)$ are now all 1
Then, we analyzed where all the 3s could be, seeing that we have a lot of 3s. Square , $(4,4)$, $(5,8)$, $(8,9)$ are now all 3.
In a similar way, we can get that $(8,7)$ and are 5 , then $(1,9)$ and$(2,5)$ is 9, and then $(9,8)$ is 2.
$(2,3)$ needs to be 8 because $(5,2)$ is 8, and then $(3,2)$ will needs to be 2.
$(3,5)$ will be 4 because there is already a 4 on $(1,8)$ so $(3,7)$ and $(3,9)$ cannot be4, then we can work out that is and $(1,7)$ is 2.



This is what the Sudoku looks like at to this point. (There is a problem with the point $(9,9)$ the three should be on $(8,9)$)

Now that it is very hard to work out other numbers, we can start to do case work. We can see that $(6,2)$ and $(8,2)$ will be a good point to start, they need to be filled with 4 and 6. We know that the correct path must be between these two possibilities.

Case1: $(6,2)$ is 4, and $(8,2)$ is 6. This case won't work because we will get a double 4s on the square $(9,3)$ and the square $(9,9)$



(case 1)

Case 2: When we set $(8,2)$ as $(8,2)$, we can easily get the values of $(8,2)$. Then, we can figure out the other blocks that's blank (the blocks with circle on are new).