**Stage 3 ★★****Mixed Selection 1 – Solutions****1. Mean Square**

Whichever number goes in the middle will need to be the mean of the most different pairs of numbers, so it must be the 5, because the 5 is in the middle of all of the numbers.

Then the numbers which go either side of the 5 must be pairs of numbers that add up to 10 (because to find the mean of 2 numbers, they must be added together and then divided by 2). So they must be:

1 and 9

2 and 8

3 and 7

4 and 6

9	6	3
8	5	2
7	4	1

9 cannot be the mean of two of the other numbers, because it is the largest number. So it must go in one of the corners.

9 is odd, and if it is added to an even number, then the result will be odd. When this odd number is divided by 2, the result will not be a whole number. So the numbers in the other corners cannot be even numbers. So they must be 3 and 7.

Filling in the remaining boxes gives the completed square.

2. A Mean Calculation

The numbers are all around 100, so the mean will be close to 100. If the numbers were all 100 smaller, then their mean would be 100 smaller. So finding the mean of the same set of numbers but all 100 smaller, and then adding 100, will give the mean.

Subtracting 100 from each of the numbers gives

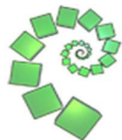
-3 -3 -2 -1 -1 -1 0 0 1 1 3 4 4 5

These numbers add up to 7, and there are 14 of them, so the mean is $7 \div 14 = 0.5$.

So the mean of the original numbers is 100.5.

A fuller solution is available at: <https://nrich.maths.org/12851/solution>

These problems are adapted from UKMT (ukmt.org.uk) and SEAMC (seamc.asia) problems.

**3. Mean Median**

We are looking for the weight of the third child, as they will have the median weight.

The total weight of all five of the children is $45 \times 5 = 225$ kg.

The weight of the lightest three children is $42 \times 3 = 126$ kg

The weight of the heaviest three is $49 \times 3 = 147$ kg

Adding the weights of the three heaviest children to the three lightest children ($126 + 147 = 273$ kg) is equal to the weight of all 5 children plus the weight of the third child, since we have counted their weight twice.

Subtracting the weight of all five children from this, we are left with the weight of just the third child, the median: $273 - 225 = 48$ kg.

4. Acceptance Rate

Over the first four years, on average 325 students are accepted, so a total of $325 \times 4 = 1300$ were accepted.

A 4% increase means that over the five year period, on average $325 \times 1.04 = 338$ students were accepted each year, so $338 \times 5 = 1690$ students were accepted in total. So in 2011, $1690 - 1300 = 390$ students were accepted.

5. Equal Means

The mean of the first three numbers is $\frac{15+5+x}{3} = \frac{20+x}{3}$.

The mean of the last four numbers is $\frac{x+7+9+17}{4} = \frac{x+33}{4}$.

As these are equal:

$$\frac{20+x}{3} = \frac{x+33}{4}$$

Multiplying both sides by 12 gives:

$$80 + 4x = 3x + 99$$

Then, subtracting $80 + 3x$ from both sides gives:

$$x = 19$$

These problems are adapted from UKMT (ukmt.org.uk) and SEAMC (seamc.asia) problems.