

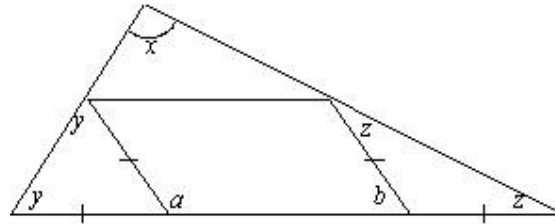


Angles, Polygons and Geometrical Proof

Stage 3 ★★★

Mixed Selection 1 – Solutions

1. Parallelogram in the middle



The two angles marked y° are equal because they are in an isosceles triangle. For the same reason, the angles z° are equal. Since an exterior angle of a triangle is the sum of the two interior and opposite angles, it follows that $a = 2y$ and $b = 2z$. Now $a^\circ + b^\circ = 180^\circ$ since they are the base angles of a parallelogram. So $2y + 2z = 180$ giving $y + z = 90$. But, from the angle sum of a triangle $x + y + z = 180$; hence $x = 90^\circ$.

2. Pentagon ring

Since each angle of a regular pentagon is 108° , each pentagon has a turn of 36° to pack against its neighbour. Therefore it will take 10 ($\times 36^\circ$) turns to complete a full-turn, i.e. 10 pentagons.

Therefore 7 more pentagons are needed.

These problems are adapted from UKMT Mathematical Challenge problems (ukmt.org.uk)