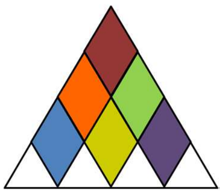
Stage 3 ★
Mixed Selection 4 - Solutions

1. Isometric rhombuses

There are a number of different ways of solving this problem.

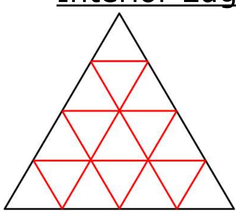
Directions



The number of vertical rhombi can be seen by looking at the possible positions of the top triangle. The following six rhombi are then apparent.
The problem has rotational symmetry, so the other two directions will also give six rhombi each.

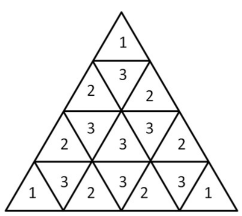
This means that the total number of rhombi is $3 \times 6 = 18$.

Interior Edges



Each rhombus that is formed has exactly one of the interior edges (marked in red) contained within it. Moreover, each interior edge corresponds to one rhombus, consisting of the triangles on either side. There are 18 interior edges, so 18 rhombi that can be formed.

Double-counting



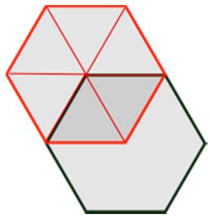
For each of the small triangles, the number of rhombi that contain it can be counted, as shown in the diagram on the right. However, this counts each rhombus twice (once for each triangle it contains). Therefore the total obtained (36) must be halved, giving a total of 18 rhombi.

2. Equilateral pair

Since VXW is an equilateral triangle, $\angle VXW = 60°$. Therefore, $\angle WXY = \angle WXV + \angle VXY = 60° + 80° = 140°$.
Since the equilateral triangles are congruent, $WX = X$, so the triangle WXY is isosceles.
Hence, $\angle YWX = \angle XYW = \frac{1}{2}(180° - \angle WXY) = \frac{1}{2}(180° - 140°) = 20°$.
Then, since VXW is equilateral, $\angle VWX = 60°$.
Then, $\angle VWY = \angle VWX - \angle YWX = 60° - 20° = 40°$.

3. Overlapping beer mats

The top beer mat, outlined in red in the diagram below, can be split into triangles from its centre, as shown below.



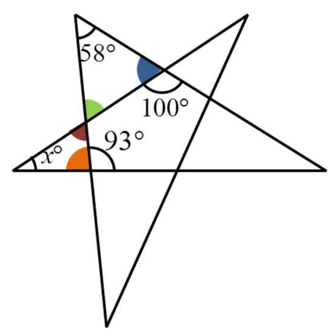
Because the hexagon is regular, the triangles are all the same, so each one will have area $6\ cm^2$ (since the area of the whole beer mat is $36 = 6 \times 6\ cm^2$).

So, since the overlap consists of 2 of the triangles, its area is $12\ cm^2$.

4. Angles please

There are two different ways of calculating the angle *x*.

The *first method* is as follows:



The blue angle can be calculated since angles on a straight line add to $180°$. This means it is $180° - 100° = 80°$.

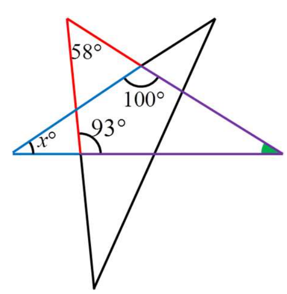
The orange angle can be calculated similarly, to be $180° - 93° = 87°$.

The green angle can then be calculated since the angles in the top-left triangle add up to $180°$. This is $180° - 80° - 58° = 42°$.

The green and red angles are opposite angles. This means that they are equal, so the red angle is also $42°$.

Then, the angles in the left-hand triangle must add up to $180°$, so $x° = 180° - 42° - 87° = 51°$.

The *other method* is by considering the farthest right angle as shown in the diagram.



The angles in the red and purple triangle must add up to $180°$, so the green angle is $180° - 58° - 93° = 29°$.

Then, the angles in the blue triangle must also add up to $180°$.

Therefore, $x° = 180° - 29° - 100° = 51°$.

These problems are adapted from UKMT (ukmt.org.uk) and SEAMC (seamc.asia) problems.