



Angles, Polygons and Geometrical Proof

Stage 3 ★

Mixed Selection 2 – Solutions

1. Robo-turn

The total angle turned through after each of the first 4 moves is $10^\circ, 30^\circ, 60^\circ$, and 100° . So the robot does not face due East at the end of a move in its first complete revolution. The total angle it has turned through after each of the next 5 moves is $150^\circ, 210^\circ, 280^\circ, 360^\circ$, and 450° , so at the end of the 9th move the robot does face East. As the robot moves 5m in each move, the distance it travels is 45m.

2. Stellar angles

The four marked angles are the interior angles of a quadrilateral. Hence, $x = 360^\circ - (105^\circ + 115^\circ + 125^\circ) = 15^\circ$.

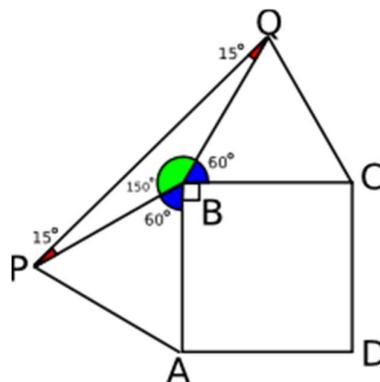
3. Two exterior triangles

Since ABQ and BCQ are equilateral, the angles ABP and CBQ are both 60° . So,
$$\angle PBQ = 360^\circ - 90^\circ - 60^\circ - 60^\circ = 150^\circ$$

PBQ is isosceles, so the angles BPQ and PQB are equal. So,

$$2 \times \angle PQB = 180^\circ - 150^\circ = 30^\circ$$

Therefore, $\angle PQB = 15^\circ$.



These problems are adapted from UKMT Mathematical Challenge problems (ukmt.org.uk)



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4. As long as possible

The length of AD must be less than 15cm , since 15cm would be its length if all four points lay in a straight line. However, by making angles ABC and BCD close to 180° , AD can be made close to 15cm in length.

As the length of AD is a whole number of centimetres, its maximum value, therefore is 14cm .

5. Polygon cradle

As $PQRST$ is a regular pentagon, each of its internal angles is 108° . The internal angles of the quadrilateral $PRST$ add up to 360° and so, by symmetry, $\angle PRS = \angle RPT = \frac{1}{2}(360^\circ - 2 \times 108^\circ) = 72^\circ$. Each interior angle of a regular hexagon is 120° , so $\angle PRU = 120^\circ$.

Therefore, $\angle SRU = \angle PRU - \angle PRS = 120^\circ - 72^\circ = 48^\circ$.

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