



# Angles, Polygons and Geometrical Proof

## Stage 3 ★

### Mixed Selection 2 – Solutions

#### 1. Robo-turn

The total angle turned through after each of the first 4 moves is  $10^\circ, 30^\circ, 60^\circ$ , and  $100^\circ$ . So the robot does not face due East at the end of a move in its first complete revolution. The total angle it has turned through after each of the next 5 moves is  $150^\circ, 210^\circ, 280^\circ, 360^\circ$ , and  $450^\circ$ , so at the end of the 9th move the robot does face East. As the robot moves 5m in each move, the distance it travels is 45m.

#### 2. Stellar angles

The four marked angles are the interior angles of a quadrilateral. Hence,  $x = 360^\circ - (105^\circ + 115^\circ + 125^\circ) = 15^\circ$ .

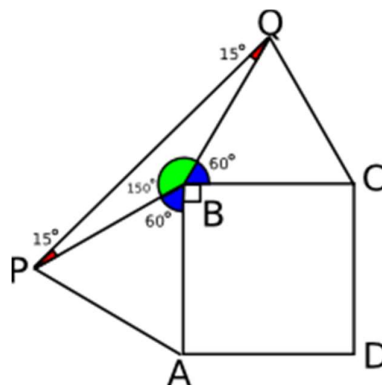
#### 3. Two exterior triangles

Since  $ABQ$  and  $BCQ$  are equilateral, the angles  $ABP$  and  $CBQ$  are both  $60^\circ$ . So,  
$$\angle PBQ = 360^\circ - 90^\circ - 60^\circ - 60^\circ = 150^\circ$$

$PBQ$  is isosceles, so the angles  $BPQ$  and  $PQB$  are equal. So,

$$2 \times \angle PQB = 180^\circ - 150^\circ = 30^\circ$$

Therefore,  $\angle PQB = 15^\circ$ .



*These problems are adapted from UKMT Mathematical Challenge problems ([ukmt.org.uk](http://ukmt.org.uk))*



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## 4. As long as possible

The length of  $AD$  must be less than  $15\text{cm}$ , since  $15\text{cm}$  would be its length if all four points lay in a straight line. However, by making angles  $ABC$  and  $BCD$  close to  $180^\circ$ ,  $AD$  can be made close to  $15\text{cm}$  in length.

As the length of  $AD$  is a whole number of centimetres, its maximum value, therefore is  $14\text{cm}$ .

## 5. Polygon cradle

As  $PQRST$  is a regular pentagon, each of its internal angles is  $108^\circ$ . The internal angles of the quadrilateral  $PRST$  add up to  $360^\circ$  and so, by symmetry,  $\angle PRS = \angle RPT = \frac{1}{2}(360^\circ - 2 \times 108^\circ) = 72^\circ$ . Each interior angle of a regular hexagon is  $120^\circ$ , so  $\angle PRU = 120^\circ$ .

Therefore,  $\angle SRU = \angle PRU - \angle PRS = 120^\circ - 72^\circ = 48^\circ$ .

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