Stage 4 ★
Mixed Selection 1 – Solutions

1. Table height

Let the block have width w, length l and the height of the table be h.
The right hand side of the table tells us: (1) $h + l = w + 96$.
The left hand side of the table tells us: (2) $h + w = 84 + l$.
$(1) + (2)$ gives $2h + (l + w) = 180 + (l + w) \Rightarrow 2h = 180 \Rightarrow h = 90$.

2. Adjacent additions

Let the 7-digit code be $abcdefg$.
We know that
$(1)\ a + b + c + d = 16$
$(2)\ b + c + d + e = 16$
$(3)\ c + d + e + f = 16$
$(4)\ d + e + f + g = 16$
$(5)\ a + b + c + d + e = 19$
$(6)\ b + c + d + e + f = 19$
$(7)\ c + d + e + f + g = 19$
If we take equation (1) away from equation (5) we obtain $e = 3$.
Similarly:
$(6) - (2)$ gives that $f = 3$,
$(7) - (3)$ gives that $g = 3$,
$(5) - (2)$ gives that $a = 3$,
$(6) - (3)$ gives that $b = 3$ and
$(7) - (4)$ gives that $c = 3$.
Then using equation (1) we find that $d = 7$.
So the code is 3337333 and the sum of the digits is 25.

These problems are adapted from UKMT Mathematical Challenge problems (ukmt.org.uk).

3. Big fish

Let the weights in kilograms of the head and body of the fish be h and b respectively. Then $h = 9 + \frac{1}{3}b$ and $b = h + 9$. So $b = 9 + \frac{1}{3}b + 9$, that is, $\frac{2}{3}b = 18$, which gives $b = 27$. Hence $h = 18$, so the whole fish weighed 54kg.

4. Carry over

UK means $10U + K$ and SMC means $100S + 10M + C$, so we have $10U + K + 4 = 100S + 10M + C$.
The left hand side is at most $10 \times 9 + 8 + 4 = 102$ so $100S + 10M + C \leq 102$. Therefore, $S \leq 1$, so $S = 1$ (since it can't be zero). So $10M + C \leq 2$. So $M = 0$ and thus M has the lowest value.

5. Sum = Product = Quotient

$b \neq 0$ since we can't divide by zero. $ab = \frac{a}{b} \Rightarrow b^2 = 1$ so $b = \pm 1$.
If $b = 1$ then the equation $a + b = ab$ becomes $a + 1 = a$ which is impossible. Hence $b = -1$, then $a + b = ab$ becomes $a - 1 = -a$ $\Rightarrow a = \frac{1}{2}$. Finally we check that $\frac{a}{b} = a + b$ holds for $a = \frac{1}{2}, b = -1$ which it does, so there is exactly one pair which satisfies the conditions.

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