Stage 3 ★
Mixed Selection 1 – Solutions

1. Shape sums

□ = 2 △, from the first equation.
Then, ◯= □ +△= 2 △ + △= 3 △
Then, ◊= ◯+ □ +△= 3 △ +2 △ + △= 6 △

Therefore, ◊ is worth 6 △.

2. Granny's age

Let my age now be x. So Granny's age is $4x$.
Five years ago, I was $x-5$ years old and Granny's age was $4x-5$. At that time, she was five times as old as I was and so:
$4x-5=5(x-5)$ giving $x=20$.

So Granny is 80 and I am 20, hence the sum of our ages now is 100.

3. Partial magic

The numbers along the leading diagonal total 58, which is therefore the sum of each row and each column. We can now calculate that the number to the left of 10 must be 20 and below that must be 7. Hence $x=21$.

4. Fifty coins

Let the number of five-pence coins be x, so the number of two-pence coins is $50-x$. Then $5x+2(50-x)=181$, that is $3x=81 \Rightarrow x=27$.

So there are 27 five-pence coins and 23 two-pence coins, so there are four more 5ps than 2ps.

These problems are adapted from UKMT Mathematical Challenge problems (ukmt.org.uk).

5. Currency exchange

In total, Ann and Dan have £1.80+£4.00=£5.80, so they need to end up with £2.90 each. Therefore, Dan needs to receive at least three five pound coins from Ann to have enough money. Then, Ann has £2.50, so she needs at least two two-pound coins from Dan, at which point they have £2.90 each. Therefore, at least 5 coins must change hands.

Alternatively, suppose Dan gives Ann x 20p coins and Ann gives Dan y 50p coins. Then Dan has $£(1.8 - 0.2x + 0.5y)$ and Ann has $£(4.0 - 0.5y + 0.2x)$.

We need $1.8 - 0.2x + 0.5y = 4 - 0.5y + 0.2x$, which is the same as $10y - 4x = 22 \Rightarrow 5y - 2x = 11$ The solution to this equation which minimises $x + y$ is $x = 2$, $y = 3$, so the smallest number of coins that must change hands is 5.

6. Average surroundings

We are given that:

$A = \frac{B+31+64+20}{4} = \frac{B+115}{4}$ and $B = \frac{A+18+54+38}{4} = \frac{A+110}{4}$

So $4A = B + 115$

$\Rightarrow 4A = \frac{A + 110}{4} + 115$

$\Rightarrow 16A = A + 110 + 460 = A + 570$

$\Rightarrow 15A = 570$

$\Rightarrow A = \frac{570}{15} = 38$

These problems are adapted from UKMT Mathematical Challenge problems (ukmt.org.uk).