Stage 3 ★★
Mixed Selection 1 Solutions

1. Seven Dwarfs

Suppose the youngest dwarf is x years old. The sum of the ages of the youngest three is $x + (x + 1) + (x + 2) = 3x + 3$, and the sum of the ages of the oldest three is $(x + 4) + (x + 5) + (x + 6) = 3x + 15$. We know that $3x + 3 = 42$, so $3x + 15 = 42 + 12 = 54$.

2. Alien currency

Let the value of a green note and the value of a blue note be g zogs and b zogs respectively. Then $3g + 8b = 46$ and $8g + 3b = 31$. Adding these two equations gives $11g + 11b = 77$, so $b + g = 7$.
Therefore $3g + 3b = 21$. Subtracting this equation from the original equations gives $5b = 25$ and $5g = 10$. So $b = 5$, $g = 2$ & $2g + 3b = 19$.

3. Symbol

$3 \oplus 5 = 3 \times 5 + 3 + 5 = 23$ and $2 \oplus x = 2x + 2 + x = 3x + 2$.
These are equal, so $3x + 2 = 23$, i.e. $x = 7$.

4. To run or not to run?

Let the athlete take x minutes to cycle one mile. Therefore he takes $\frac{3x}{2}$ minutes to run one mile and $3x$ minutes to walk one mile. Now, $3x + \frac{3x}{2} + x = 3x + 10 \Rightarrow x = 4$. The cyclist takes 12 minutes to walk the first mile, 6 minutes to run the second mile and 4 minutes to cycle the third mile, a total time of 22 minutes.

These problems are adapted from UKMT Mathematical Challenge problems (ukmt.org.uk).

5. Fractions of fractions

$\frac{2}{3}\times\frac{5}{6}\times X=\frac{3}{4}\times\frac{4}{5}\times Y \Rightarrow \frac{5X}{9}=\frac{3Y}{5}$

$25X=27Y, so\ \frac{X}{Y}=\frac{27}{25}$

6. Packing boxes

In one hour, Harry and Christine together pack 18 boxes. Harry and Betty together pack 12 boxes. Christine and Betty together pack 9 boxes. Therefore, Harry packs 3 more boxes than Christine, looking at the rates when they're each with Betty. In one hour Christine would therefore pack 7.5 boxes.