

Introduction to the Lorentz Transformation

$\gamma = 1/\sqrt{1 - v^2/c^2}$ The Lorentz Contraction

this is gamma – the third letter of the Greek alphabet

v = speed of the object (in m/s)

m/s is often written as ms^{-1}

c = speed of light
300,000,000 m/s
which is 3×10^8 m/s

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This allows us to calculate the observed length, time and mass of an object when it is travelling close to the speed of light. You need to calculate γ for a scenario and then apply it to the relevant relativistic equation.

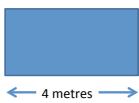
$\gamma = 1/\sqrt{1 - v^2/c^2}$ The Lorentz Contraction

Length contraction states that the original Length (L_o) of an object gets smaller, according to an external (stationary) observer, when travelling close to the speed of light (c). This new length is L .

$$L = L_o / \gamma$$

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Jim has a problem. His garage is 4 metres long, but his ladder is 5 metres. The ladder won't fit!
[We will assume Jim isn't clued up enough to consider putting it diagonally ...]



← 4 metres →

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Jim realises that, to a stationary observer, his ladder will shrink if he runs very fast with it. He decides to calculate how fast he will need to run so the observed length of his 5m ladder becomes 4m and it will therefore fit into his garage.

[He doesn't, unfortunately, consider what will happen a moment later, but hey – science can be dangerous!]

The Lorentz Contraction

$$1 = 5/\gamma$$

$$4 = 5 \times \sqrt{1 - v^2/c^2}$$

$$4/5 = \sqrt{1 - v^2/c^2}$$

$$6/25 = 1 - v^2/c^2$$

$$\gamma = 1/\sqrt{1 - v^2/c^2}$$

$$L = L_0/\gamma$$

So: $L_0 = 5$ metres and $L = 4$ metres

This means Jim's ladder will fit if he runs at $3/5$ of the speed of light.

$$v^2/c^2 = 1 - 16/25$$

$$v^2/c^2 = 9/25$$

$$v/c = 3/5$$

$$v = 3/5 c$$

The Lorentz Contraction

Length contraction compares the original length L_0 and the new length L .

Time dilation compares T_0 (time on clock at rest) with the new, slower time T .

Mass increase compares M_0 (original mass at rest) with the new, increased mass M .