Improving Reasoning: Analysing Different Approaches

Article by Malcolm Swan

If you look carefully at the November 2011 NRICH problems (http://nrich.maths.org/thismonth/all/2011/11) you will notice a slight change in emphasis. NRICH has always encouraged mathematical reasoning through interesting and imaginative problems, but this month we are giving much more emphasis to the *analysis* of reasoning.

Everyone agrees that it is important to develop reasoning in mathematics lessons. It is a main objective of mathematics curricula around the world[1], but the evidence from most classrooms seems to show that reasoning is insufficiently developed[2]. Why should this be?

After many years watching and designing mathematics lessons, it has become clear to me that most of the attention is on 'answer-getting', not on improving the quality of the reasoning. As soon as pupils get the answer, they are moved onto a fresh problem, they don't reflect much on the process they went through. Reasoning is not improved because it does not become the *object of attention*. Most reasoning remains invisible it stays inside people's heads. In order for students to improve their reasoning, it needs to be made visible and audible through oral or written explanations.

Recently, at Nottingham, we have been trying out a model for the design of 'formative' mathematics lessons that try to address this issue. This approach encourages pupils to stick with a problem and revise and refine their solutions over two lessons. The plan is as follows:

First lesson:

Introduce a challenging problem to the class and allow pupils to have a go at it, individually, without help. The intention here is to allow pupils time to engage with the problem and to explore some possible approaches. Most pupils may not get very far and will need reassuring that this doesn't matter.
Collect in pupils' initial ideas and look through them. Don't mark the work, but try to get an overview of pupils' ideas and existing ways of thinking. Prepare a few oral questions that you can give to move their thinking forward. These questions should not be about 'getting pupils to an answer', but rather about deepening their thinking. So 'can you think of a more helpful representation?' is better than 'why don't you try drawing a graph?'.

Second lesson:

Re-issue the problem and allow pupils time to work in pairs, sharing their different approaches. Where necessary, prompt pupils with the prepared questions.

Allow pupils time to have a second go at the problem producing a joint attempt. If this part is produced on a poster, a few pairs can be asked to share their thinking with the whole class. Encourage questions and suggestions from other pupils as they do this.
Give each pair of pupils some written examples of other pupils' reasoning ('sample work'). These should show a range of possible approaches. For example, they may show a numerical, a graphical and an algebraic approach. These examples shouldn't be complete or perfect, because you want pupils to critique and improve them.
Finally, ask pupils to work together to produce an elegant, complete, polished solution.

We have tried this approach now in many classrooms with a wide range of problems. What we have found is that the quality of the solutions improves quite dramatically. What is even more remarkable though, is how much pupils' reasoning develops.

The role of the sample work seems to be a key factor in this. In concept-focused lessons, the sample work may be chosen to illustrate common mistakes and misconceptions. In problem-solving lessons the sample work may focus on alternative approaches. In each case the sample work makes the reasoning visible. An important feature of problem solving is that pupils are given an opportunity to select and apply *any method they choose* from their mathematical 'toolkit'. But this leads to a dilemma:

> Given the choice, pupils only use mathematics with which they are comfortable, like trial and error. They don't choose algebra! They never realise the potential of more sophisticated methods.
> If they are told which methods to use, then problems degenerate into technical exercises and pupils never learn to use mathematics autonomously.

This dilemma can be resolved using the sample work. After having a go at the problem themselves, pupils are confronted with a series of potential approaches that show ways of thinking that contrast with their own. They are encouraged to engage with each piece of reasoning, first to comprehend it, then to improve and complete it. Pupils develop critical skills during this process. Compared with teacher-provided methods, sample pupil reasoning has less status, and pupils enjoy challenging it.

When we tried out this approach with teachers, some used authentic samples of work taken from their own classroom, while others created 'fake' sample work designed to illustrate particular learning points. Some commented that social issues interfered when the reasoning came from other pupils in the same class - pupils became less critical when commenting on work produced by friends. In our work we

prefer to base the work on authentic examples, but rewrite them to make them anonymous.

In this month's NRICH problems there are many opportunities for pupils to analyse reasoning in the sample approaches. In Lots of Lollies, for example, after pupils have had time to try the problem themselves, they are given two partial solutions, one *numerical* and one *diagrammatic*, and are asked to think how these could be extended to form a solution. In Fitted, a *practical* approach is also included. In Steel Cables, four different correct approaches are shown that lead to the same solution. Here the challenge is to make sense of each solution and decide on a 'favourite'. (Actually my favourite isn't there - it involves seeing the figure as the difference of two cubes!). This could lead to discussion of such issues as 'elegance'. In Odds, Evens and More Evens, three starting points are given, each of which involves seeing the sequences of numbers in different ways.

I hope you will try to engage pupils with reasoning not just with these problems, but with many problems of your own.

Finally, how often do we let pupils see us reasoning? One final challenge then! Choose a problem from the NRICH website *that you have never seen before.* Try to tackle it with a class, thinking aloud. Try different approaches. Ask the class to help with ideas when you get stuck. Go down blind alleys. If you get nowhere, then park the problem overnight and return to it later. If students are to learn what thinking like a mathematician is like, let them hear your thinking!

At Nottingham, we are working on an EU project called PRIMAS (Promoting Inquiry in Mathematics and Science across Europe). The videos and materials being developed for this project will contain some of these ideas, linked to classroom videos. They may be found at: http://primas.mathshell.org.uk/pd.htm. This work is not yet finished, so watch the site for changes in the near future!

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[1] E.g. 'Mathematics equips pupils with a uniquely powerful set of tools to understand and change the world. These tools include logical reasoning, problem solving skills, and the ability to think in abstract ways.' (Mathematics national curriculum, QCA 1999)
[2] E.g. 'Pupils rarely investigate open-ended problems which might offer them opportunities to choose which approach to adopt or to reason and generalise. Most lessons do not emphasise mathematical talk enough; as a result, pupils struggle to express and develop their thinking.' (Understanding the score, Ofsted, 2008)