$$y = \frac{10}{x} \rightarrow xy = 10$$

Therefore, because length multiplied by width gives area, every point represents a rectangle of area 10.

$$y = \frac{10}{x} \rightarrow x = \frac{10}{y}$$

Therefore, for every point (m, n) on the curve, $n = \frac{10}{m}$, so $m = \frac{10}{n}$, so (n, m) is also a point on the curve. Since (n, m) is the reflection of (m, n) in the line $y = x$, the graph is symmetrical about the line $y = x$. This can also be thought of as the rectangles having a line of symmetry such that if the dimensions x and y are swapped around, the rectangle is still the same.

As $x \rightarrow \infty$, $\frac{10}{x} \rightarrow 0$, so $y \rightarrow 0$, so the curve approaches the line $y = 0$ (i.e. the x-axis). However, it never touches the x-axis, since infinity is not a number, such that x can be equal to it, so y never actually equals 0. Again, this can be worked out by considering the rectangles represented by the points. As we have seen, each has an area of 10, which clearly would not be possible if one of the dimensions was 0.

$$y = \frac{5}{x} = \frac{1}{2} \times \frac{10}{x}$$

Therefore every y-coordinate has half the value of the corresponding y-coordinate of the first graph, so the graph is effectively half as high. The curves won't intersect due to the y-coordinates being different for each value, even when x is very large and the lines *appear* to touch. Another way of thinking about this is that each point on this graph represents a rectangle of area 5 (by the same argument as before). If the curves were to intersect at a point, that point would represent both a rectangle of area 10 and a rectangle of area 5, which is clearly a contradiction.

If the line $y = \frac{1}{2}P - x$ and curve $y = \frac{10}{x}$ intersect for all values of P, it would mean that for any perimeter there exists a rectangle with dimensions x and y such that the area is 10. Considering that the maximum area for a given perimeter is given by a square, a rectangle with perimeter 4 has maximum area 1 (when it is a square of side length 1). This counterexample shows that the line and curve don't intersect for all P. To find the smallest possible perimeter of a rectangle, we need to find the smallest value of P such that the line and curve intersect, which will be when the line is a tangent to the curve. To find this, we can form two simultaneous equations, use them to produce a quadratic with P as a coefficient of x, and find the value of P that will give a repeated root for the quadratic, so that the line and curve only touch. This is achieved as follows:

$$y = \frac{1}{2}P - x$$

$$y = \frac{10}{x}$$

$$\frac{1}{2}P - x = \frac{10}{x}$$

$$\frac{x}{2}P - x^2 = 10$$

$$x^2 - \frac{1}{2}P + 10 = 0$$

To find the value of P that gives a repeated root, we want the discriminant to equal 0.

$$\left(\frac{1}{2}P\right)^2 - 4 \times 1 \times 10 = 0$$

$$\frac{1}{4}P^2 = 40$$

$$P^2 = 160$$

$$P = 4\sqrt{10}$$

This result could be determined more easily, if it is accepted that the maximum area for a given perimeter is given by a square, as I stated earlier. The minimum perimeter to have area 10 must have 10 as its maximum area, so, with the side length of a square perimeter P being $\frac{P}{4}$:

$$\left(\frac{P}{4}\right)^2 = 10$$

$$\frac{P^2}{16} = 10$$

$$P^2 = 160$$

$$P = 4\sqrt{10}$$