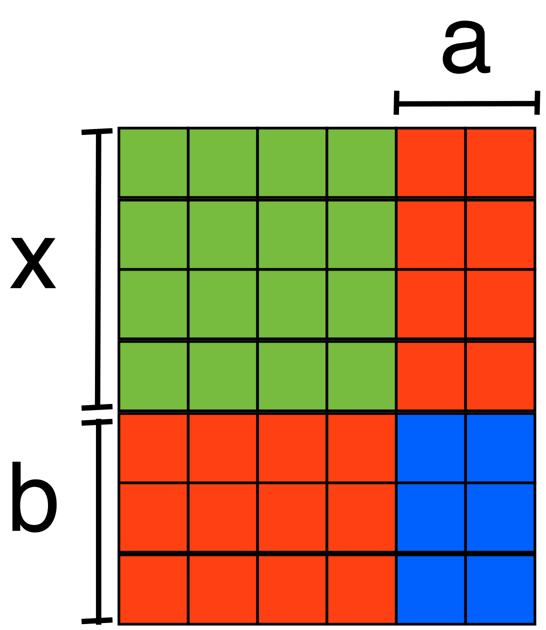
Consider the generic rectangle:



-The green section is a square, so its area is equal to x^2

-The red section consists of two rectangles with dimensions a, x and b, x. Therefore the red area is equal to ax+bx, which equals x(a+b)

-We can see that the area of the blue section will always have dimensions a and b, so its area is equal to ab, if it 'fills' the gap created by the red area.

-The total area is equal to the sum of these component areas.

Thus in a rectangle that 'works': **$A = [x^2] + [x(a+b)] + [ab]$**

So for the rectangle '$x^2 + 7x + 12$', we can see that 1) 7=a+b and 2) 12=ab.

a=3; b=4 satisfies these equations.

Thus for all values of x (i.e. all bases): a=3; b=4 will yield the rectangle '$x^2 + 7x + 12$', since **$A = (x+3)(x+4) = x^2 + 7x + 12$**

Using one square and 12 sticks:

x^2 coefficient is 1;

a+b=12

Therefore: $A = x^2 + 12x + ab$

Again this will be satisfied for all x values (i.e. in any base)

With one square and 100 sticks:

Let the total width = W; and the total height = H:

W=x+a; H=x+b

Therefore W+H = 2x + a+b
And because a+b=100: W+H = 2x + 100

A = WH = (x+a)(x+b) which multiplies out to give $A = x^2 + x(a+b) + ab$. And because a+b = 100, it follows that $A = x^2 + 100x + ab$

Using one square and 12 units:
x^2 coefficient is 1;
ab = 12

W=x+a; H=x+b
Therefore W+H= 2x + a+b

A = (x+a)(x+b), yielding: $A = x^2 + x(a+b) + ab$. Because ab=12, it follows that
$A = x^2 + x(a+b) + 12$

With one square and 100 units:
By similar arguments, $A = x^2 + x(a+b) + 100$

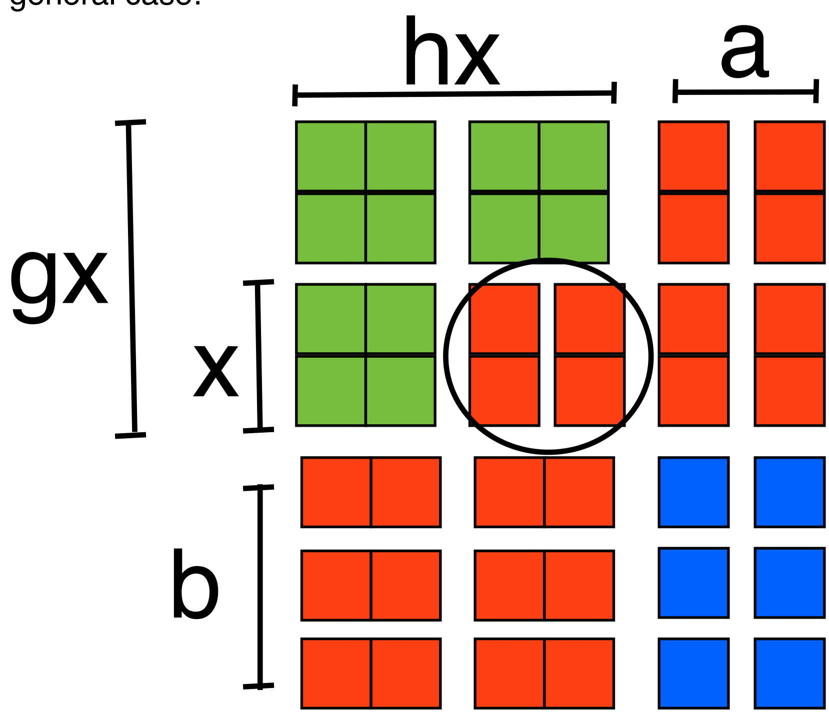
Using p sticks and q units:
x^2 coefficient is 1;
a+b = p;
ab = q.

Since W=x+a; H=x+b:
W+H = 2x + a+b,
Therefore W+H = 2x + p

Since $A = x^2 + px + q$:
Substituting in our values for p & q gives **$A = [x^2] + [x(a+b)] + [ab]$**

However, we never defined what our x value could be. It therefore follows that these width, height and area equations hold true for all values of x, so long as you use only one square.

Thus far we have only considered rectangles with one 'x^2' (or square). So let's consider a more general case:



Immediately this poses some problems. Although we can still calculate the blue area such that it 'fills' the gap the red area creates (ab), how can we find out the red or green areas, or even the width or height of the rectangle?

Let the number of squares vertically be g, and horizontally be h.

-The height of the green area is then gx, and the width hx.

If we take the circled red area as part of the green area, the green area becomes ghx^2, and the red area becomes bhx+agx (so it will always 'fill' the gap that the green area creates).

-Thus **$A = ghx^2 + (bh+ag)x + ab$**

W and H are easy to find:

W = hx + a;
H = gx + b;
Therefore W+H = x(g+h) + a + b

Again we can see that if this is true we will get a rectangle, since the red area will account for the 'gap' that the green area creates, and the blue area will always do likewise for the red area. Thus what x value (base) we use or even how many squares we use, will not pose a problem

So we could put any values in for the constants a, b, g, and h, and the resulting equation will describe a rectangle that will 'work' for any base.