8th March 2011  Vector Journeys  Niharika Paul

\[ A = (3, 1) \]
\[ B = (-1, 3) \]
\[ C = -A = (-3, -1) \]
\[ D = -B = (1, -3) \]

Another route:

He starts his journey by walking along \[ \vec{A} = (1) \]

\[ B = (-1, 1) \]
\[ C = -A = (-1, 1) \]
\[ D = (1, 1) \]

Another route:

\[ A = (1, 0) \]
\[ B = (0, 0) \]
\[ C = (0, -1) \]
\[ D = (0, -1) \]

Once you know \( A \), you can determine \( B \), \( C \), \( D \). \( A \) has an opposite direction to \( C \) vector and \( |A| = |C| \).

\( B \) is 90° to \( A \) and \( |B| = |A| \). So we know \( B \).

\( \vec{B} \) and \( D \) have opposite directions and \( |B| = |D| \). So we know \( D \).
I have noticed the following pattern between $A$, $B$, $C$, $D$

If $A = (x_1, y_1)$, $C = (-x_1, -y_1)$

Also, if $A \perp B$, then I conjecture that to get $B$ from $A$ you swap the coordinates of $A$ and change the sign of one component: $B = (-y_1, x_1)$ or $(-x_1, y_1)$

$D = (y_1, x_1)$ or $(-y_1, x_1)$ respectively

This pattern $\Rightarrow$ if $B = (x_2, y_2)$ then,

$$(x_1, x_2) + (y_1, y_2) = 0$$

In Fig 1 and Fig 2, Alison's path is marked in pink.
In Fig 1, $E = (0)$. In Fig 2, $E = (1)$.

We realise $E = A + B$

Using Pythagoras

$$|E|^2 = |A|^2 + |B|^2$$

If Charlie's path is a square,

$$|E|^2 = 2|A|^2$$

and $E$ is $45^\circ$ to $A$

We can know $E$ given $A$

Here's how $A + B = E$

$$(x_1 + x_2, y_1 + y_2) = (\frac{2}{4})$$

$$(x_1 + x_2) = 2 = 1$$

$$(y_1 + y_2) = 4 = 2$$

$B = (x_2, -y_1)$ or $(-x_1, y_1)$
In the first case, or \(-y_1 = x_2\) and \(y_2 = x_1\),

\[
\begin{align*}
x_1 - y_1 &= 2 \\
y_1 + x_1 &= 4
\end{align*}
\]

\[\text{In the 2nd case:} \quad \begin{align*}
x_1 + y_1 &= 2 \\
y_1 - x_1 &= 4
\end{align*}\]

... \(x_1 = 3\), \(x_2 = 3\), \(y_1 = 3\), \(y_2 = 1\), \(A = (\frac{1}{3})\), \(B = (\frac{1}{3})\).

Alison's journey can be uniquely defined by a given A on the grid. You measure 45° and draw a line.