The three edge numbers are \( a, b, c \). Choose a pair of factors of \( a \); let \( a = xy \). Let us see if one of the factors is common with \( b \); let \( x \) be common factor of \( a \) and \( b \). Then in the vertex between \( a \) and \( b \) write \( x \). In the vertex between \( a \) and \( c \) write \( y \) but if \( y \) is not a common factor of \( a, c \) then the pair of factors \( x, y \) doesn't work. Then you have to repeat choice of factors of \( a \). If \( y \) is common to \( a, c \) then write \( y = z \). If \( x \neq z \) then the pair of factors \( x, y \) does not work. Then you move on to the next pair of factors of \( a \).

\[
\begin{align*}
\frac{\text{xyz}}{\text{abc}} &= \frac{x y z}{a b c} \\
&= \frac{x y z}{a b c} \\
&= \frac{x y z}{a b c} \\
&= \frac{x y z}{a b c} \\
\end{align*}
\]

Product of edge numbers is always the square of product of vertex numbers.
Q3. Case 1: Only edge number changing is a.

\[ a \text{ is scaled by a factor of } n, \quad n \in \mathbb{N}, \quad n > 0 \]

The scale by which \( x \) and \( y \) have to be multiplied has to be the same so that \( z \) can compensate for the scaling.

The square root of \( x \) and \( y \) is \( \sqrt{n} \). This is why \( n \) should be positive.

The scaling of \( z \) should be \( \frac{1}{\sqrt{n}} \).

Case 2: Only edge numbers changing are \( a \) and \( b \).

\[ a \quad \text{and} \quad b \quad \text{are scaled by a factor of } n \quad \text{each, } \quad n \in \mathbb{N}, \quad n > 0. \]

\[ y \quad \text{is scaled by a factor of } i, \quad i > 0. \]

\[ x \quad \text{is scaled by a factor of } k, \quad k > 0. \]

\[ z \quad \text{is scaled by a factor of } i, \quad i > 0. \]

\[ \frac{g \cdot k}{i} = n \quad \quad \text{(1)} \]

\[ \frac{g}{i} = n \quad \quad \text{(2)} \]

\[ i = 1 \quad \quad \text{(3)} \]

\[ \frac{g}{i} = 1 \quad \quad \text{(4)} \]

\( \frac{g}{i} \times 3 \quad \Rightarrow \]

\[ g = 1 \]

\[ \frac{g \cdot k}{i} = n \]

\[ \frac{k}{i} = n \]

\[ i = 1 \]

Only \( x \) is scaled by a factor of \( n \).

Case 3: When \( b \) and \( c \) are all scaled by a factor of \( n \)

\[ x, y, z \quad \text{all scaled by } \sqrt{n} \]

Q4. The only arithmetic group possible is

\[ \frac{a}{x}, \quad b, \quad c, \quad \in \mathbb{Z} \]

\[ y = n, \quad n > 1, \quad x = mn, \quad m \in \mathbb{Z} \]

\[ z = ln, \quad \alpha \in \mathbb{Z} \]

\[ \triangle \]