Q1

Here we choose the sides of the square as vectors.

Let: \( A = (x, y) \)

Then \( B = (y, -x) \) or \( (y, -x) \)

\( C = -A = (-x, -y) \)

\( D = -B = (x, y) \) or \( (-y, x) \)

So we see for a given \( A \) there are 2 possibilities for a square.

Q2

If \( A \) joins the vertices \( Z \) and \( Y \) which are \( x \), \( y \), \( x', y' \), \( x'' \), \( y'' \) in \( Z \).

We want to prove that, if \( X \equiv (x_1, y_1) \) and \( W \equiv (x_2, y_2) \)

Then \( x_3, y_3 \), \( x_4, y_4 \) \( x, y \in \mathbb{Z} \)

Now,

\( (x) = (x_2 - x_1) \)

or \( (y) = (y_2 - y_1) \)

Now \( B = (-y) \) or \( (y, -x) \)

Case 1: \( B = (y) = (x_3 - x_2) \)

Now, \( x_2, y_1, y_2, y_3, y_4 \) \( x, y \in \mathbb{Z} \)

\( \Rightarrow x, y \in \mathbb{Z} \)
or \( y = x_2 - x_2 \) and \( x = y_2 - y_2 \)
\( x, y \in \mathbb{Z} \) and \( x_2, y_2 \in \mathbb{Z} \)

This also implies for \( x = x_4 \) and \( y_4 \), and case 2.

Given:

\[ |A1| = |B| \]

\( E \) at 45° to \( A \) and \( B \)

\[ E = A + B \]

\[ E = (x) \quad A = (x_1) \quad B = (x_2) \]

\[ (x) = (x_1) + (x_2) \]

\[ (y) = (y_1) + (y_2) \]

\[ B = (-y_1) \text{ or } (y_1) \]

\[ B = (-y_1) \]

Then \( (x) = (x_1 + y_1) \)

or \( x = x_1 + y_1 \) and \( y = y_1 + x_1 \)

or \( x = y_1 \) and \( y = x_1 + y_1 \)

or \( x = y_1 \) and \( y = x_1 + y_1 \)

or \( 2y = y - x \)
or \( y_1 = \frac{y - x}{2} \)

\( x_1 = \frac{x + y}{2} \)

Dr. We know \( x, y \) are integers. However, \( x \) and \( y \) will not always be integer-valued. It will be integer-valued when \( y - x \) and \( x + y \) are even.

Charlie is right because there is no fixed angle in a rhombus’ corner. So there are 0 possibilities for the rhombus ABCD.

He are going to construct a circle using A as the centre and AB as the radius. The number of points on the circle that lie on grid points are the number of rhombuses that have these vertices on grid points.

Yes I agree with Alison.
We are going to draw another straight line at 90° to AB, and then check if C and D are grid points. Then CD is the other diagonal of the rhombus ABCD.