I can extend Charlie's table to find more sets of Pythagorean Triples where the hypotenuse is 1 unit longer than one of the other sides?

Sets of Pythagorean Triples

The squares   The full sum

$3^2+4^2=5^2$	9+16=25
$5^2+12^2=13^2$	25+144=169
$7^2+24^2=25^2$	49+576=625
$9^2+40^2=41^2$	81+1600=1681
$11^2+60^2=61^2$	121+3600=3721
$13^2+ 84^2=85^2$	169+7056=7225
$15^2+ 112^2=113^2$	225+12544=12769
$17^2+144^2+145^2$	289+20736=20825

I notice the following pattern:-

The smallest and the largest side of any right-angled triangle is an odd number and the second smallest side of any right-angled triangle is an even number.
The square of the smallest side of a right-angled triangle is one less than double the largest side of a right-angled triangle.

I can make the following predictions:-

I predict that the smallest number in a right-angled triangle is always an odd number when the hypotenuse is one more than the second smallest side.

I can find a formula that generates Pythagorean Triples like Charlie's:-

If you have a right-angled triangle with sides n, s and h where $2a+1=n$ and $s+1=h$,the formula is:
$(n^2-1)/2= s$
$(n^2+1)/2=h$

a=any integer
n=smallest side in units
s=second smallest side in units
h=hypotenuse

I can prove that my formula works-

If you take 7 you would get 24 and 25 by doing this:
$7^2=49$ 49-1=48 48/2=24

$7^2=49$ 49+1=50 50/2=25

$7^2+24^2=25^2$
When expanded, you get this:
49+576=625
You could do this with any number for example with 5 $5^2+1=26$ 26/2=13
$5^2-1=24$ 24/2=12

25+144=169

Alison's problem-2 units

Alison has been working on Pythagorean Triples where the hypotenuse is 2 units longer than one of the other sides.
So far, she has found these:

$$4^2+3^2=5^2$$

$$6^2+8^2=10^2$$

$$8^2+15^2=17^2$$

I can find more Pythagorean Triples like Alison's

Below is a table of Pythagorean Triples where the hypotenuse is 2 units longer than one of the other sides.

$10^2+24^2=26^2$	100+576=676
$12^2+35^2=37^2$	144+1225=1369
$14^2+48^2=50^2$	196+2304=2500
$16^2+63^2=65^2$	256+3969=4225
$18^2+80^2=82^2$	324+6400=6724
$20^2+99^2=101^2$	400+9801=10201
$22^2+120^2=122^2$	484+14400=14884
$24^2+143^2=145^2$	576+20449=21025

I can find a formula for generating Pythagorean Triples like Alison's:-

If we had a right-angled triangle with sides n, s and h where 2a+2=n and h-s=2, the formula would be:
$n^2/4 -1=s$
$n^2/4 +1=h$

a=any integer
n=smallest side in units
s=second smallest side in units
h=hypotenuse

I can prove that my formula works :-

If you take 22 you would get 120 and 122 by doing this:
$22^2=484$ 484/4=121 121-1=120

$22^2=484$ 484/4=121 121+1=122

When expanded, you get this:
484+14400=14884
You could do this with any number for example with 14
$14^2=196$ 196/4=49 49-1=48
$14^2=196$ 196/4=49 49+1=50
196+2304=2500

Here are some follow-up questions you might like to consider:

I can find Triples where the hypotenuse is 3 units longer than one of the other sides? Or 4 units longer? Or...-

You can find Triples if you just know the difference between the hypotenuse and one of the other sides so where the hypotenuse is so many units longer than one of the other sides by using this formula:

When a right angled triangle has sides n, s and h you could use this formula where
a(d)+ (d)(d-1)= n so that:
$n^2- (d)^2/2d=s$

$n^2-(d)^2/2(d)+d=h$

a=any integer
d=difference between the hypotenuse and one of the other sides.
n=smallest side in units
s=second smallest side in units
h=hypotenuse